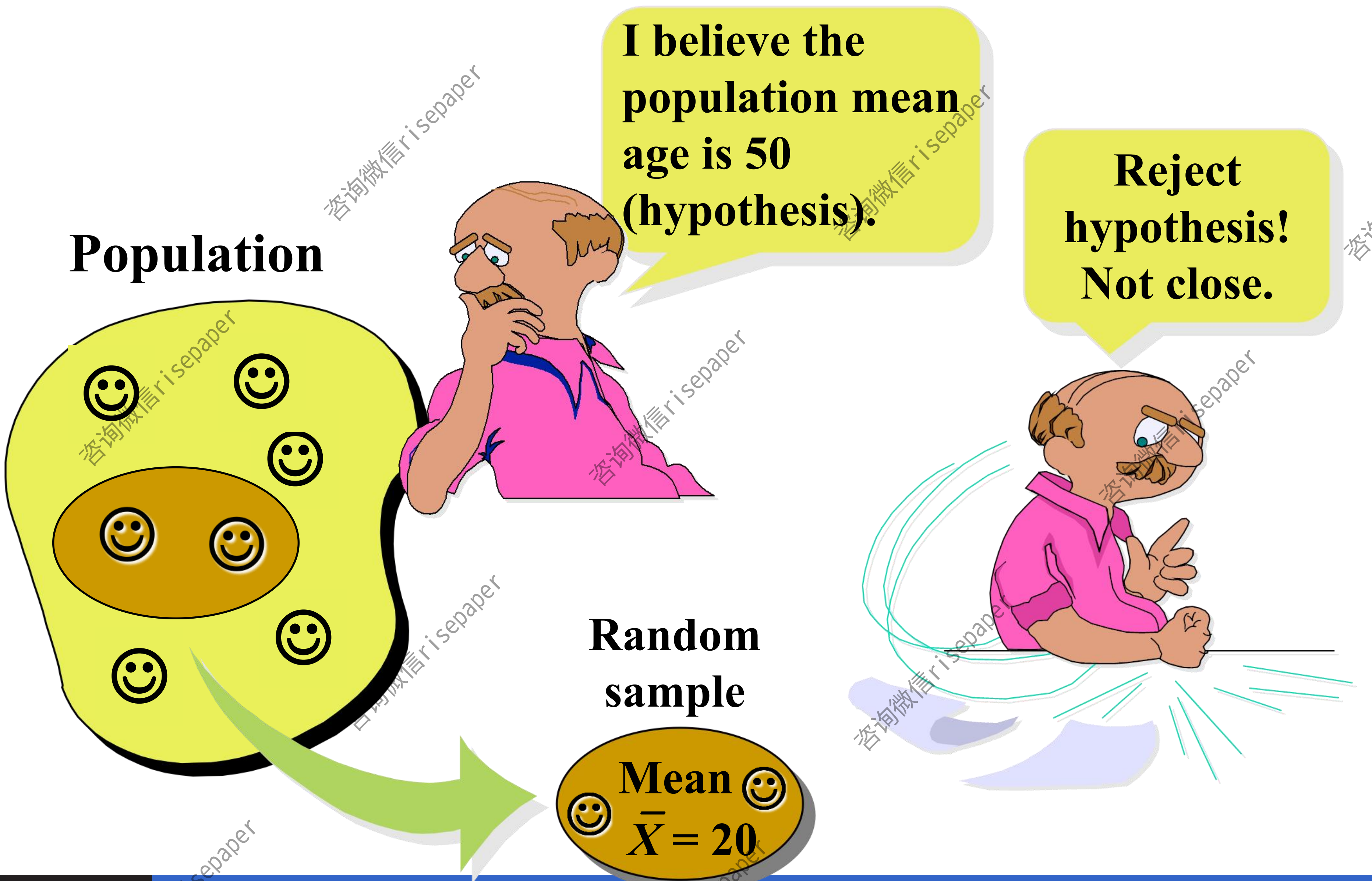
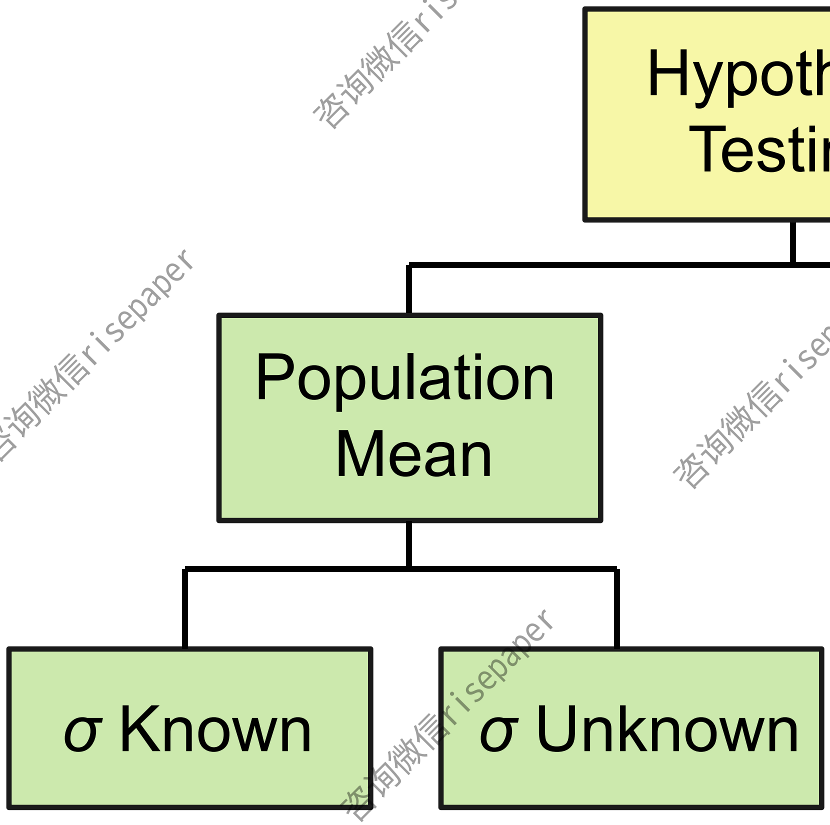
What's a Hypothesis?

A statistical **hypothesis** is a statement about the numerical value of a population parameter.

Hypothesis Testing



Hypothesis Testing for Single Populations



Null Hypothesis

The **null hypothesis**, denoted H_0, represents the hypothesis that will be accepted unless the data provide convincing evidence that it is false. This usually represents the "status quo" or some claim about the population parameter that the researcher wants to test.

Alternative Hypothesis

The **alternative (research) hypothesis**, denoted H_a, represents the hypothesis that will be accepted only if the data provide convincing evidence of its truth. This usually represents the values of a population parameter for which the researcher wants to gather evidence to support.

An Introduction to Hypothesis Testing

A **hypothesis** is an assumption about a population parameter such as a mean or a proportion

Example: population mean

- The mean data use for smartphone users is $\mu = 1.8$ gigabytes per month

Example: population proportion

- The proportion of cell phone users with 4G contracts is $p = 0.62$

Stating the Hypothesis

Every hypothesis test has both a **null hypothesis** and an **alternative hypothesis**

The **null hypothesis (*H_0*)** represents the status quo

- States a belief that the population parameter is ≤, =, or ≥ a specific value
- The null hypothesis is believed to be true unless there is overwhelming evidence to the contrary

The **alternative hypothesis (*H_1*)** represents the opposite of the null hypothesis

- Is believed to be true if the null hypothesis is found to be false
- The alternative hypothesis always states that the population parameter is >, ≠, or < a specific value

Stating the Hypothesis

The purpose of hypothesis statements is to draw a conclusion about an unknown population parameter

- A hypothesis test is often performed to show that a change has occurred from the status quo
- The alternative hypothesis is used to represent the claim researchers wish to support
- Because of this, the alternative hypothesis is also known as the **research hypothesis**

H_0: the unknown parameter has not changed from the status quo

H_1: there has been a change in the desired direction

What Are the Hypotheses?

Is the population average amount of TV viewing 12 hours?

State the question statistically: $\mu = 12$

State the opposite statistically: $\mu \neq 12$

Select the alternative hypothesis: H_a: $\mu \neq 12$

State the null hypothesis: H_0: $\mu = 12$

What Are the Hypotheses?

Is the population average amount of TV viewing *different* from 12 hours?

State the question statistically: $\mu \neq 12$

State the opposite statistically: $\mu = 12$

Select the alternative hypothesis: H_a: $\mu \neq 12$

State the null hypothesis: H_0: $\mu = 12$

What Are the Hypotheses?

Is the average cost per hat less than or equal to $20?

State the question statistically: $\mu \leq 20$

State the opposite statistically: $\mu > 20$

Select the alternative hypothesis: H_a: $\mu > 20$

State the null hypothesis: H_0: $\mu = 20$

Identifying Hypotheses

Example problem: Test that the population mean is not 3

Steps:

- State the question statistically ($\mu \neq 3$)
- State the opposite statistically ($\mu = 3$)
  - Must be mutually exclusive & exhaustive
- Select the alternative hypothesis ($\mu \neq 3$)
  - Has the ≠, **<**, or **>** sign
- State the null hypothesis ($\mu = 3$)

It is always about a population parameter, not about a sample statistic

Sample evidence is used to assess the probability that the claim about the population parameter is true

A. It starts with Null Hypothesis, H_0

$$H_0: \mu = 3 \quad \text{and} \quad \bar{X} = 2.79$$

1. We begin with the assumption that H_0 is true and any difference between the sample statistic and true population parameter is **due to chance** and not a real (systematic) difference.
2. Similar to the notion of "innocent until proven guilty"
3. That is, "innocence" is a null hypothesis.

Stating the Hypothesis

Stating the null and alternative hypotheses depends on the nature of the test and the motivation of the person conducting it

$H_0: \mu \le 1.8$
$H_1: \mu > 1.8$

This test would be used by someone who thinks that data use has gone up, and wants to support that the average data use is now more than 1.8 gigabytes per month

$H_0: \mu \ge 1.8$
$H_1: \mu < 1.8$

This would be used by someone who wants to test an assumption that data use has gone down (rejecting the null hypothesis would support the alternative that the average data use is less than 1.8 gigabytes per month)

$H_0: \mu = 1.8$
$H_1: \mu \ne 1.8$

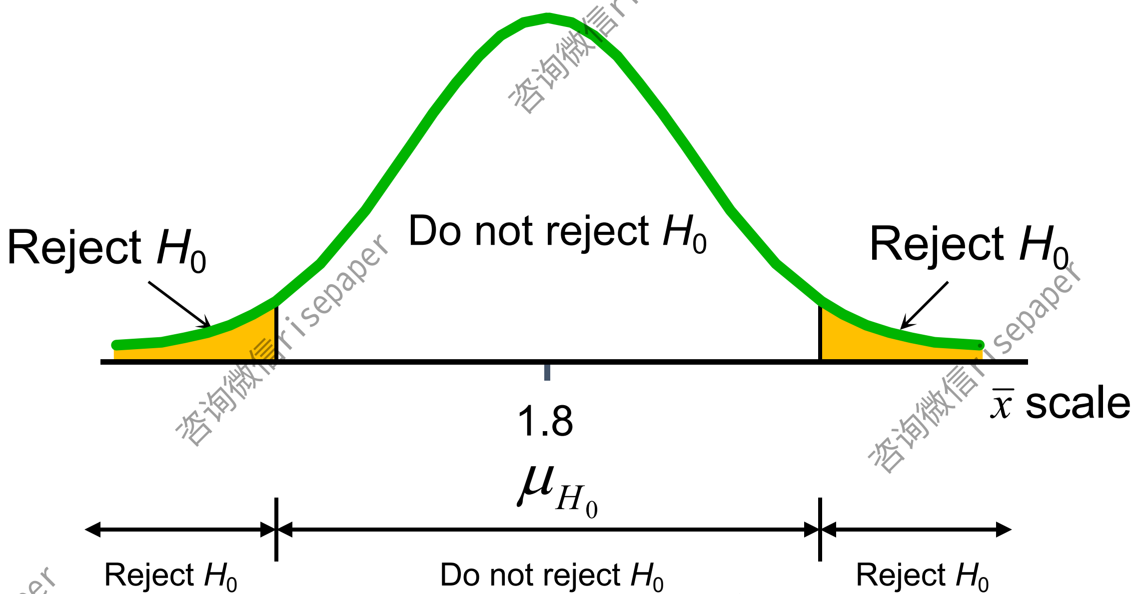
This test would be used by someone who has no specific expectation, but wants to test the assumption that the average data use is 1.8 gigabytes per month

Two-Tail Hypothesis Tests

A **two-tail hypothesis test** is used whenever the alternative hypothesis is expressed as ≠

H_0: μ = 1.8
H_1: μ ≠ 1.8

We assume that μ = 1.8 unless the sample mean is much higher or much lower than 1.8

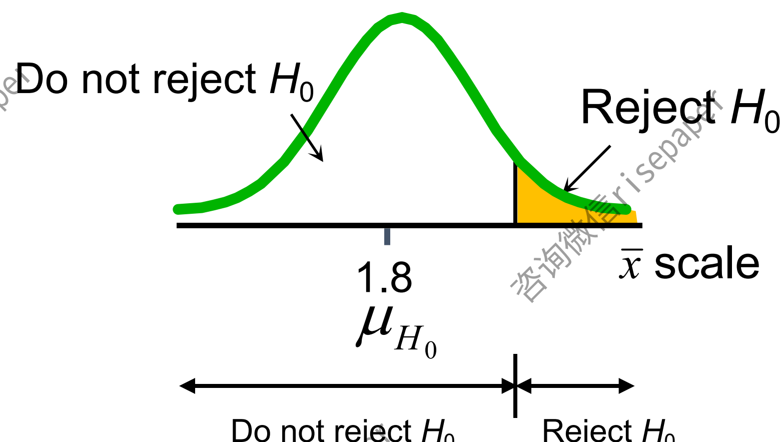


One-Tail Hypothesis Tests

A **one-tail hypothesis test** is used when the alternative hypothesis is stated as < or >

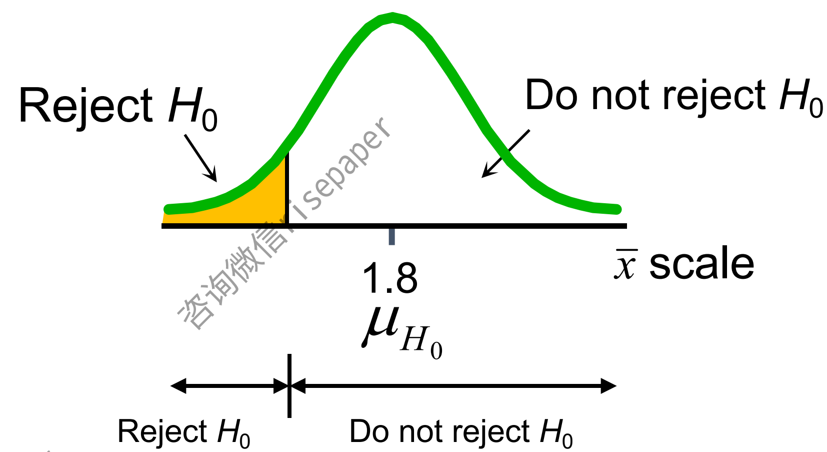
H_0: $\mu \leq 1.8$
H_1: $\mu > 1.8$

Upper tail test: We assume that μ = 1.8 unless the sample mean is much higher than 1.8



H_0: $\mu \geq 1.8$
H_1: $\mu < 1.8$

Lower tail test: We assume that μ = 1.8 unless the sample mean is much lower than 1.8



The Logic of Hypothesis Testing

The null hypothesis can never be accepted

The only two options available are to

(1) **reject** the null hypothesis, or

(2) **fail to reject** the null hypothesis

The null hypothesis is tested using sample data

- The sample result provides enough evidence to reject the null or does not provide enough evidence to reject

The Difference Between Type I and Type II Errors

Sample evidence is not perfect due to sampling error, so a conclusion about the population can be wrong

A **Type I error** occurs when the null hypothesis is rejected when it is true

- The probability of making a Type I error is known as α , the level of significance

A **Type II error** occurs when we fail to reject the null hypothesis when it is not true

- The probability of making a Type II error is known as β

Statistical Error

Sometimes H_0 will be rejected (based on large test statistic & small P value) even though H_0 is really true

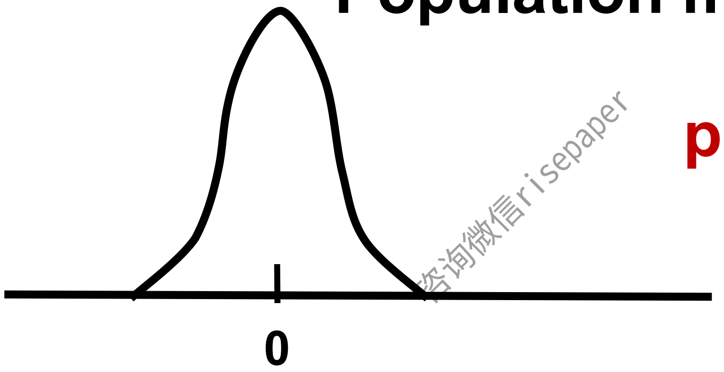
i.e., if you had been able to measure the entire population, not a sample, you would have found no difference between and μ some value- but based on Xbar you see a difference.

The mistake of rejecting a true H_0 will happen with frequency α

So, if H_0 is true, it will be rejected ~5% of the time as α frequently = 0.05

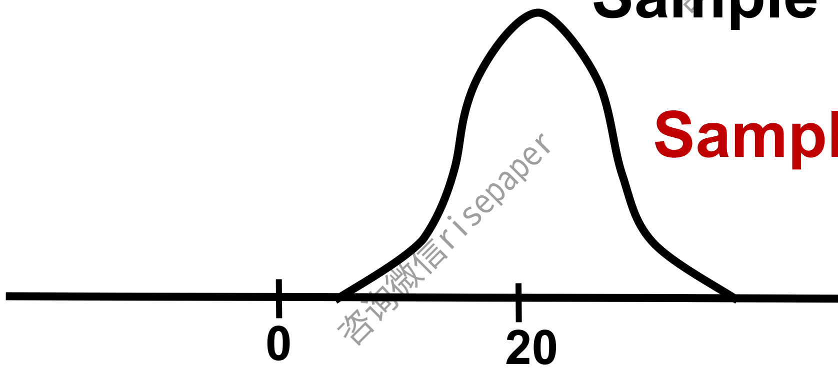
H_0 : mean = 0

Population mean = 0



population="True"

Sample mean = 20



Sample=What you see

Conclude based on sample mean that population mean ≠ 0, but it really "is" (H_0 true), therefore you have falsely rejected H_0

Type 1 Error

Statistical Error

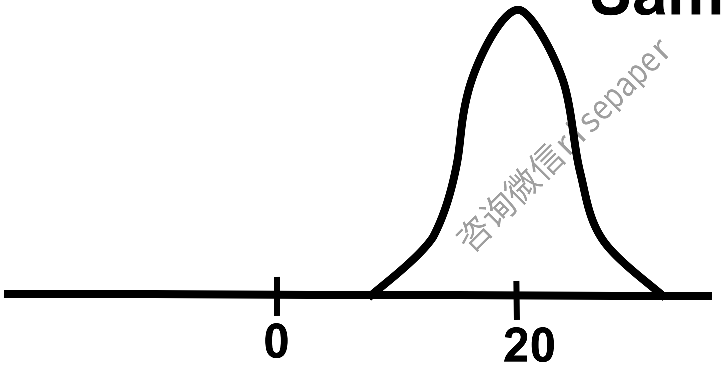
Sometimes H_0 will be accepted (based on small test statistic & large P value) even though H_0 is really false

i.e., if you had been able to measure the entire population, not a sample, you would have found a difference between and μ some value- but based on Xbar you do not see a difference.

The mistake of accepting a false H_0 will happen with frequency β

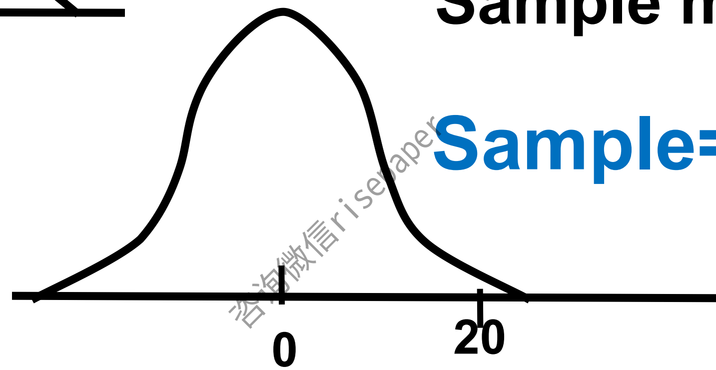
H_0 : mean = 0

Sample mean = 20



Population= "True"

Sample mean = 0



Sample= what you see

Conclude based on sample mean that population mean = 0, but it really does not (H_0 really false), therefore you have falsely failed to reject H_0

Type 2 Error

Finicky Words

Reject the null hypothesis (or other)

Fail to reject the null hypothesis

Statistically correct

Accept the null hypothesis

Support the null hypothesis

I think these are OK

~~**Prove** the null hypothesis to be true~~

The Difference Between Type I and Type II Errors

Decision Rules for the Two Types of Hypothesis Errors

Possible Hypothesis Test Outcomes

	Actual State of H_0	
Decision	H_0 is True	H_0 is False
Reject H_0	Type I Error P(Type I Error) = α	Correct Outcome
Do Not Reject H_0	Correct Outcome	Type II Error P(Type II Error) = β

The Difference Between Type I and Type II Errors

Consider a manufacturing or quality control setting:

H_0: process is satisfactory

H_1: process is not satisfactory

A Type I error is known as the **producer's risk**

- when it occurs the producer is looking for a problem in its process that does not exist

A Type II error is known as the **consumer's risk**

- when it occurs the customer is getting a product from a process that is not performing properly

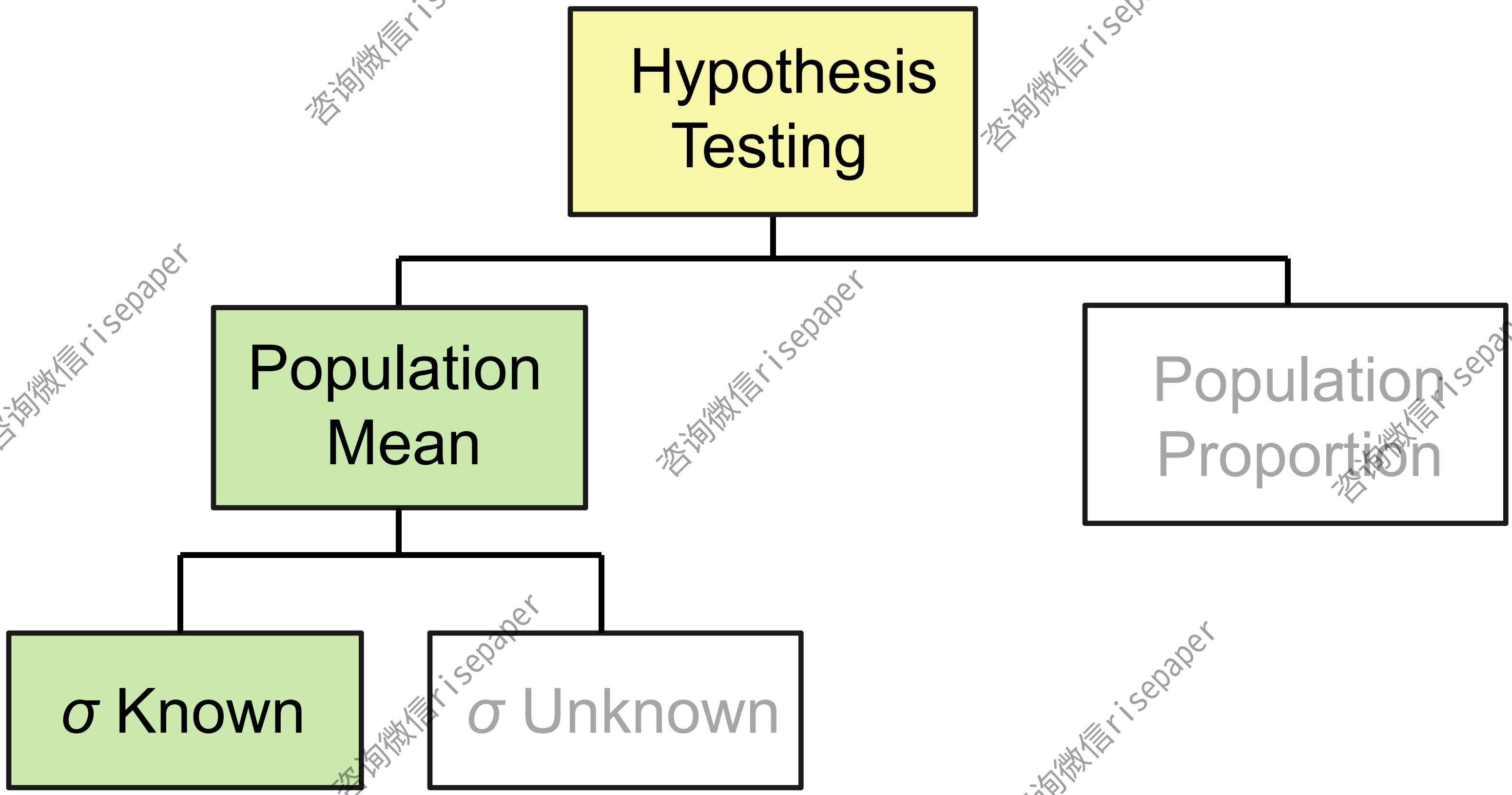
The Difference Between Type I and Type II Errors

When doing a hypothesis test, decide on a value for α **before** selecting the sample

Once α has been set, the value of β can be calculated

- For a given sample size, reducing the value of α will result in an increase in the value of β
  (or the opposite, $\alpha \uparrow \rightarrow \beta \downarrow$)
- The only way to reduce both α and β simultaneously is to increase the sample size

Hypothesis Testing for the Population Mean when σ is Known



Hypothesis Testing for the Population Mean when σ is Known

Hypothesis testing when σ is known:

1. If the sample size is small ($n < 30$) the population must follow the normal distribution

2. If the sample size is large ($n \geq 30$) the Central Limit Theorem states that the sampling distribution follows the normal distribution, so there is no restriction on the population distribution

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{\text{observed mean} - \text{expected mean}}{\text{std error}}$$

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: A new CFL bulb is claimed to have an average life that exceeds 8,000 hours

- Suppose the average life of a random sample of 36 of the new bulbs is 8,120 hours
- Assume that the population standard deviation for the life of CFL bulbs is 500 hours

The following slides show the steps to complete this hypothesis test

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

Step 1: Identify the null and alternative hypotheses

H_0: $\mu \le 8{,}000$ hours	(status quo: average life is not greater than 8,000 hours)
H_1: $\mu > 8{,}000$ hours	(the new bulb does last longer than 8,000 hours)

Step 2: Set a value for the significance level, α

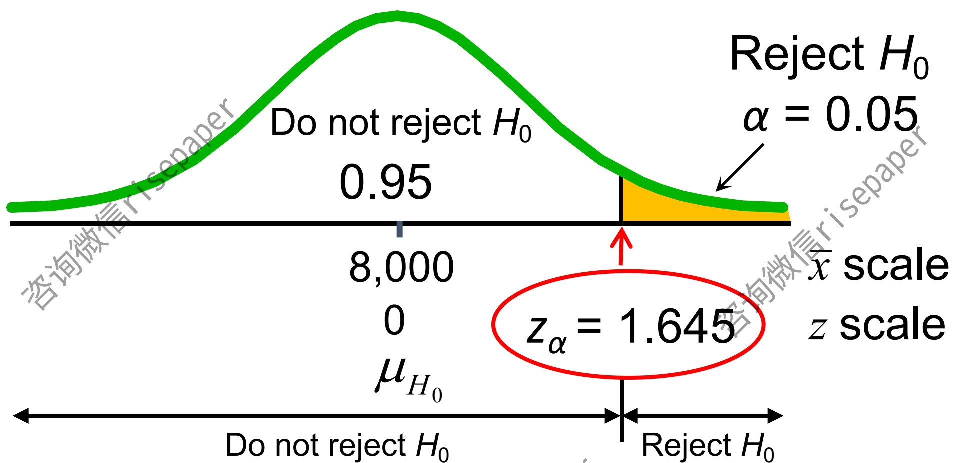
- The level of significance represents the probability of making a Type I error
- Suppose that $\alpha = 0.05$ is chosen

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

Step 3: Determine the appropriate **critical value**

- σ is known so use a *z*-score; the **critical *z*-score** identifies the rejection region for this one-tail test
- Since this is a one-tail test the entire area for α = 0.05 is placed on the right side (upper tail) of the sampling distribution:



An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

Step 4: Calculate the appropriate test statistic

Formula for the *z*-test statistic for a hypothesis test for the population mean (when σ is known):

$$z_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{\sigma}{\sqrt{n}}}$$

where:

$z_{\bar{x}}$ = The *z*-test statistic

$\bar{x}$ = The sample mean

μ_{H_0} = The mean of the sampling distribution, which is assumed to be true for the null hypothesis

σ = The standard deviation of the population

n = The sample size

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

Step 4: Calculate the appropriate test statistic

$$z_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{\sigma}{\sqrt{n}}} = \frac{8{,}120 - 8{,}000}{\frac{500}{\sqrt{36}}} = \frac{120}{83.33} = 1.44$$

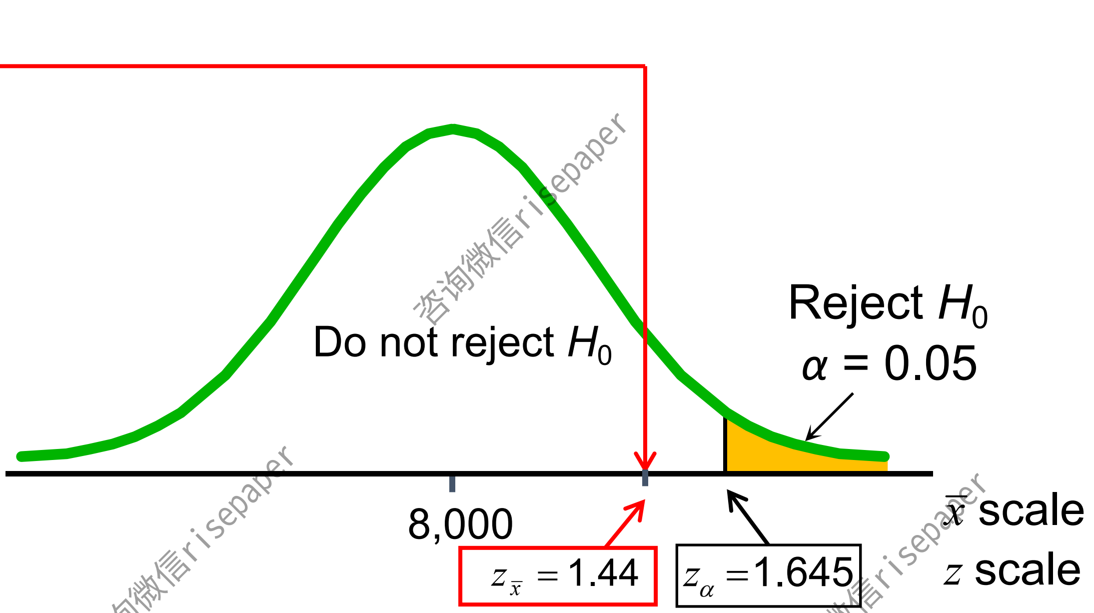
Step 5: Compare the z-test statistic $z_{\bar{x}}$ with the critical z-score z_{α}

- For a one-tail upper tail test, reject the null hypothesis if $z_{\bar{x}} > z_{\alpha}$
- Here, 1.44 is not greater than 1.645, so **do not reject H_0**
- (Illustrated on the next slide)

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Step 5: (continued)

$z_{\bar{x}} = 1.44$ is not greater than $z_{\alpha} = 1.645$, so **do not reject H_0**



An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Known)

TABLE 9.3 | Decision Rules for Hypothesis Tests When Comparing the z-Test Statistic ($z_{\bar{x}}$) with the Critical z-Score (z_α)

TEST	HYPOTHESIS	CONDITION		CONCLUSION
Two-tail	$H_0: \mu = \mu_0$	$\lvert z_{\bar{x}} \rvert > \lvert z_{\alpha/2} \rvert$	$\rightarrow$	Reject H_0
	$H_1: \mu \neq \mu_0$	$\lvert z_{\bar{x}} \rvert \leq \lvert z_{\alpha/2} \rvert$	$\rightarrow$	Do not reject H_0
One-tail	$H_0: \mu \leq \mu_0$	$z_{\bar{x}} > z_\alpha$	$\rightarrow$	Reject H_0
(upper)	$H_1: \mu > \mu_0$	$z_{\bar{x}} \leq z_\alpha$	$\rightarrow$	Do not reject H_0
One-tail	$H_0: \mu \geq \mu_0$	$z_{\bar{x}} < -z_\alpha$	$\rightarrow$	Reject H_0
(lower)	$H_1: \mu < \mu_0$	$z_{\bar{x}} \geq -z_\alpha$	$\rightarrow$	Do not reject H_0

Rule for this example (applies to the One-tail (upper) rows)

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

Step 6: (Final Step) State the conclusion

(State a "real world" conclusion.)

- According to our sample of 36 new CFL bulbs, there is not enough evidence to support the claim that the average life of these bulbs exceeds 8,000 hours.

Rejection Regions

	Alternative Hypotheses		
	Lower-Tailed	Upper-Tailed	Two-Tailed
$\alpha = .10$	$z < -1.282$	$z > 1.282$	$z < -1.645$ or $z > 1.645$
$\alpha = .05$	$z < -1.645$	$z > 1.645$	$z < -1.96$ or $z > 1.96$
$\alpha = .01$	$z < -2.326$	$z > 2.326$	$z < -2.575$ or $z > 2.575$

The *p*-value Approach to Hypothesis Testing: One-Tail Tests

The ***p*-value** is the probability of observing a sample mean at least as extreme as the one selected for the hypothesis test, assuming the null hypothesis is true

The *p*-value is sometimes referred to as the **observed level of significance**

Provides another approach to deciding whether or not to reject the null hypothesis

The p-value Approach to Hypothesis Testing: One-Tail Tests

Use the CFL bulb example to illustrate:

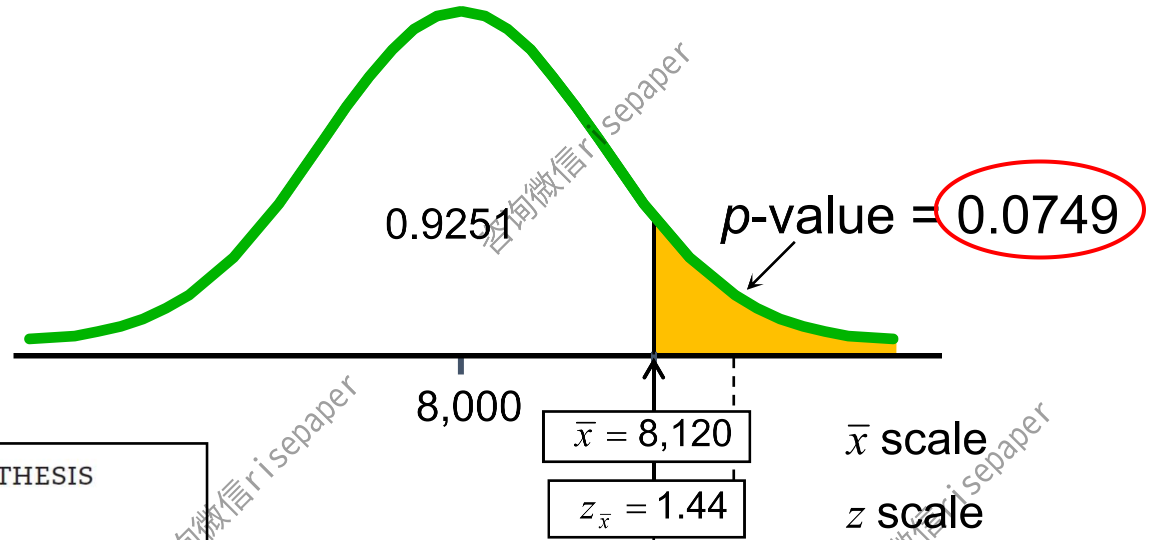
- Claim: $\mu \geq 8{,}000$
- Sample result: $n = 36$, $\bar{x} = 8{,}120$
- $\sigma = 500$ was assumed known

The p-value represents the probability of obtaining a sample mean of 8,120 hours or greater if the true population mean is 8,000 hours

=1-NORM.S.DIST(1.44,TRUE) =0.0749

$$P(\bar{x} > 8{,}120) = P\left(z_{\bar{x}} > \frac{8.120 - 8{,}000}{\frac{500}{\sqrt{36}}}\right) = \mathrm{P}(z_{\bar{x}} > 1.44) = 1 - 0.9251 = 0.0749$$

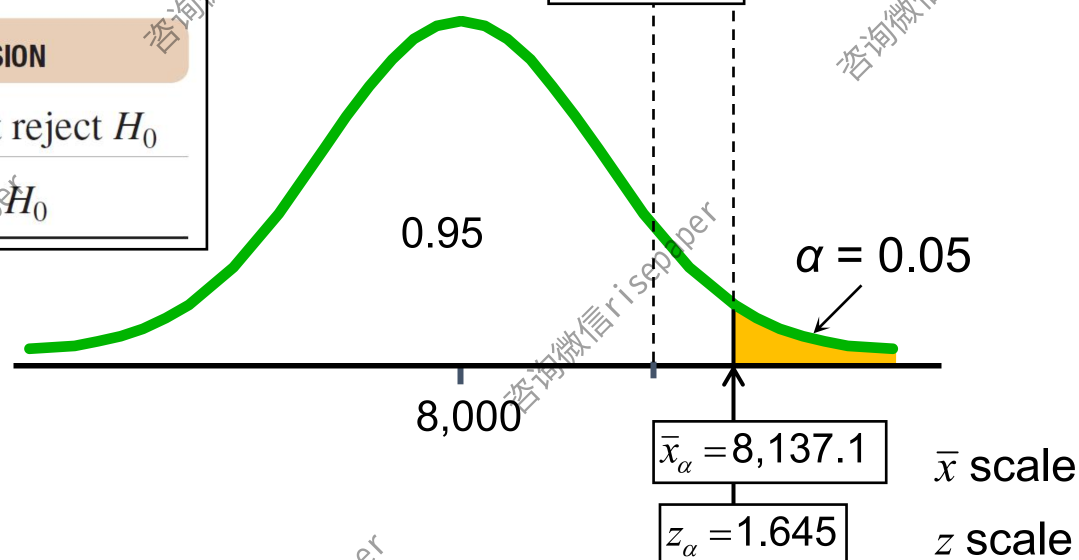
The p-value Approach to Hypothesis Testing: One-Tail Tests



Compare the p-value with α:

TABLE 9.4 | Decision Rules for Hypothesis Tests Using the p-Value

CONDITION		CONCLUSION
p-Value $\geq \alpha$	$\rightarrow$	Do not reject H_0
p-Value $< \alpha$	$\rightarrow$	Reject H_0



Since 0.0749 > 0.05,

we **do not reject H_0**

The *p*-value Approach to Hypothesis Testing: One-Tail Tests

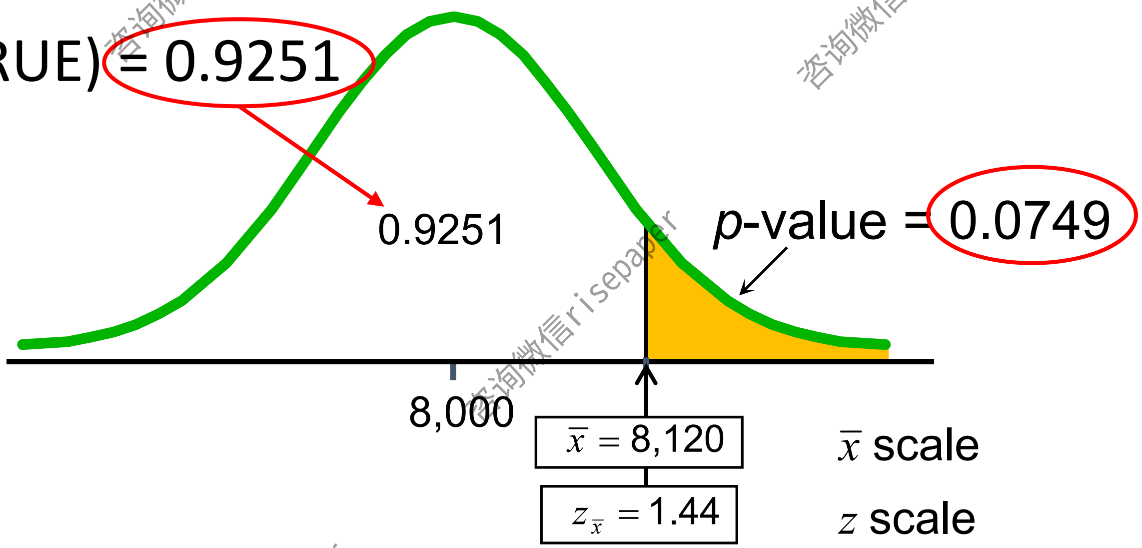
Excel's **NORM.S.DIST** function can also be used to determine the p-value

=NORM.S.DIST(*z*, cumulative)

Input the test statistic value for z and set cumulative = TRUE to get the area to the left of the test statistic:

=NORM.S.DIST(1.44, TRUE) = 0.9251

Because our *p*-value is the area to the right of the test statistic, we subtract 0.9251 from 1.0



An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: The mean data use for smartphone users is claimed to be μ = 1.8 gigabytes per month

- Suppose data use is recorded for 49 randomly selected smartphone users and the average use is found to be 1.86 gigabytes per month
- Assume that the population standard deviation is 0.20 gigabytes per month

When do we use a two-tail test? when do we use a one-tail test?

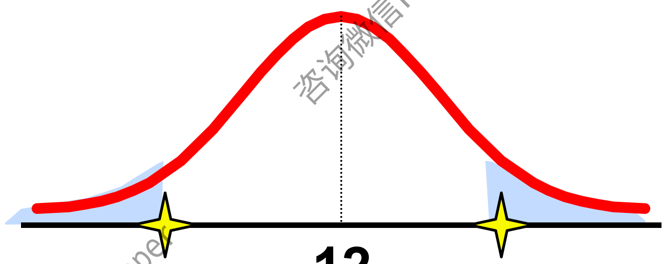
The answer depends on the question you are trying to answer.

A two-tail is used when the researcher has no idea which direction the study will go, interested in both direction. (example: testing a new technique, a new product, a new theory and we don't know the direction)

A new machine is producing 12 fluid once can of soft drink. The quality control manager is concern with cans containing too much or too little. Then, the test is a two-tailed test. That is the two rejection regions in tails is most likely (higher probability) to provide evidence of H_1.

$H_0 : \mu = 12\ oz$

$H_1 : \mu \neq 12\ oz$



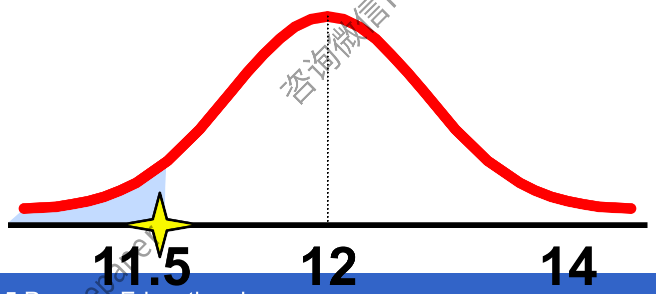
One-tail test is used when the researcher is interested in the direction.

Example: The soft-drink company puts a label on cans claiming they contain 12 oz. A consumer advocate desires to test this statement. She would assume that each can contains at least 12 oz and tries to find evidence to the contrary. That is, she examines the evidence for less than 12 0z.

What tail of the distribution is the most logical (higher probability) to find that evidence? The only way to reject the claim is to get evidence of less than 12 oz, left tail.

$$H_0 : \mu \geq 12 \text{ oz}$$

$$H_1 : \mu < 12 \text{ oz}$$



An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

Step 1: Identify the null and alternative hypotheses

$H_0: \mu = 1.8$	(status quo: average use is 1.8 gigabytes per month)
$H_1: \mu \neq 1.8$	(average use is not equal to 1.8 gigabytes per month)

Step 2: Set a value for the significance level, α

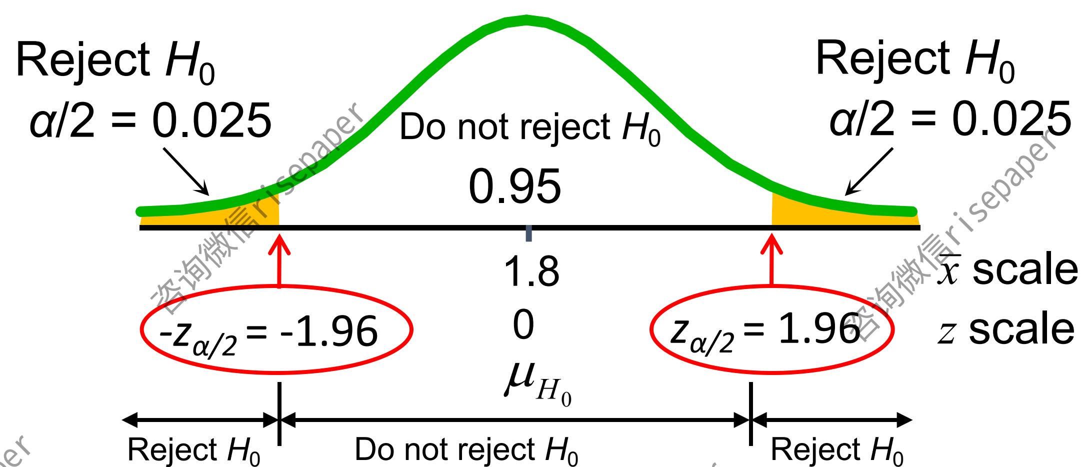
- The level of significance represents the probability of making a Type I error, chosen before conducting the hypothesis test
- Suppose that $\alpha = 0.05$ is chosen

An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

Step 3: Determine the appropriate **critical values**

- σ is known so use a *z*-score; the **critical *z*-score** identifies the rejection region for this one-tail test
- Since this is a two-tail test, $\alpha = 0.05$ is split evenly into two tails:



An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

Step 4: Calculate the appropriate test statistic

$$z_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{\sigma}{\sqrt{n}}} = \frac{1.86 - 1.8}{\frac{0.20}{\sqrt{49}}} = \frac{0.06}{0.0286} = 2.10$$

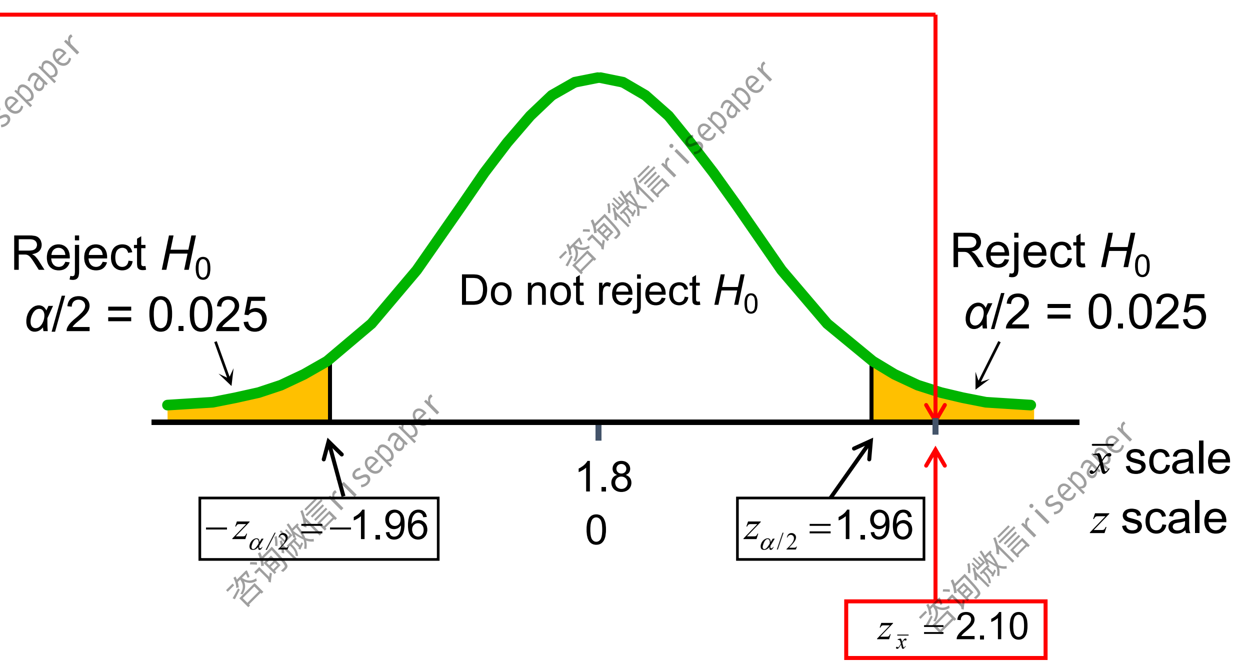
Step 5: Compare the *z*-test statistic $z_{\bar{x}}$ with the critical *z*-score $z_{\alpha/2}$

- For a two-tail test, reject the null hypothesis if $|z_{\bar{x}}| > |z_{\alpha/2}|$
- Here, 2.10 is greater than 1.96, so **reject *H*$_0$**
- (Illustrated on the next slide)

An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Step 5: (continued)

$z_{\bar{x}} = 2.10$ is greater than $z_{\alpha/2} = 1.96$, so **reject H_0**



An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Known)

Example: (continued)

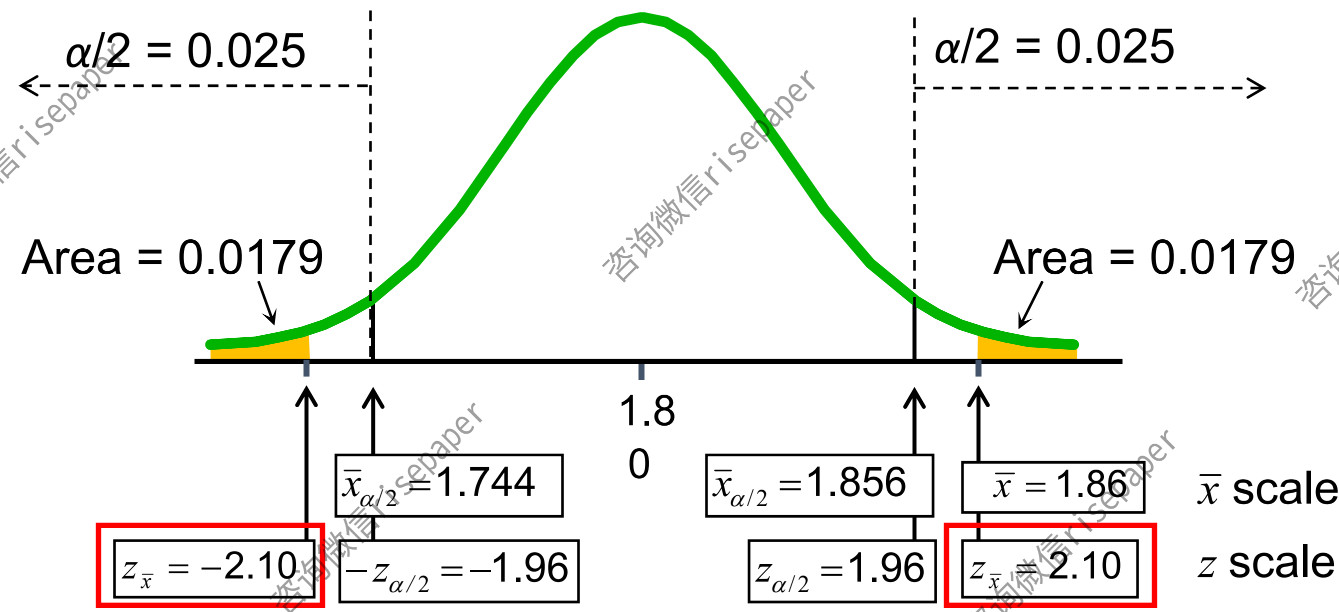
Step 6: (Final Step) State the conclusion

- Our sample of 49 smartphone users provides sufficient evidence to reject the null hypothesis, so we support the alternative hypothesis that the average data use is not equal to 1.8 gigabytes per month.

The *p*-value Approach to Hypothesis Testing: Two-Tail Tests

For a two-tail hypothesis test, the *p*-value is a sum of two tail areas

- Illustrate using the smartphone example:



$$p\text{-value} = 2 \times P(\bar{x} > 1.86) = 2 \times P(z_{\bar{x}} > 2.10) = 2(0.0179) = 0.0358$$

The *p*-value Approach to Hypothesis Testing: Two-Tail Tests

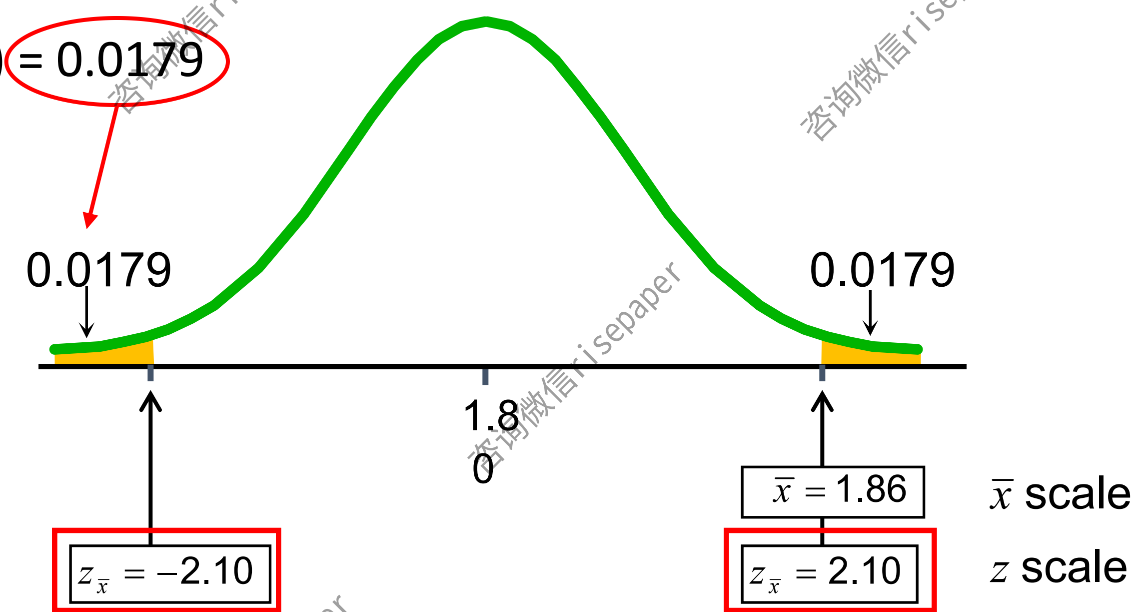
Excel's **NORM.S.DIST** function can be used to determine the p-value

=NORM.S.DIST(z, cumulative)

Input z = -2.11 to get the desired area in the lower tail, then multiply by 2

=NORM.S.DIST(-2.11, TRUE) = 0.0179

Because we need the area in both tails, we multiply 0.0179 by 2 to get a *p*-value of 0.0358



Test of Hypothesis for the Mean (σ Known)

- Convert sample result ($\bar{x}$) to a z value

Hypothesis Tests for μ

σ Known | **σ Unknown**

Consider the test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$

(Assume the population is normal)

The decision rule is:

$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > z_\alpha$$

p-Value Approach to Testing

- p-value: Probability of obtaining a test statistic more extreme (≤ or ≥) than the observed sample value given H_0 is true
  - Also called observed level of significance
  - Smallest value of α for which H_0 can be rejected

Example: Upper-Tail Z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over \$52 per month. The company wishes to test this claim. (Assume $\sigma = 10$ is known)

Form hypothesis test:

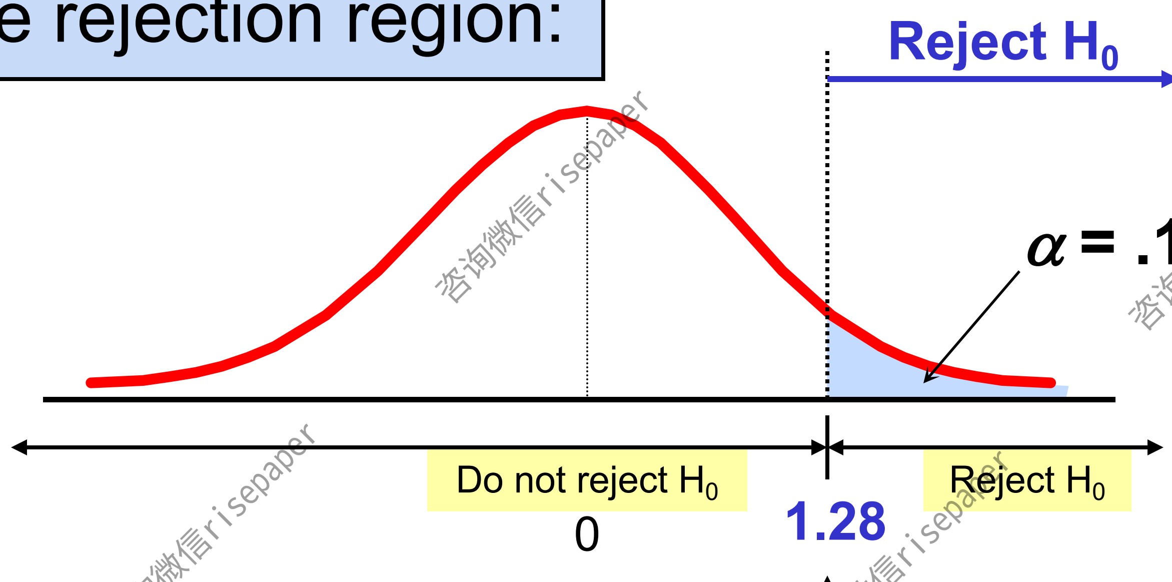
H_0: $\mu \le 52$	the average is not over \$52 per month
H_1: $\mu > 52$	the average **is** greater than \$52 per month (i.e., sufficient evidence exists to support the manager's claim)

Example: Find Rejection Region

(continued)

- Suppose that α = .10 is chosen for this test

Find the rejection region:



$$\text{Reject } H_0 \text{ if } \quad z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} > 1.28$$

Example: Sample Results

(continued)

Obtain sample and compute the test statistic

Suppose a sample is taken with the following results: $n = 64$, $\bar{x} = 53.1$ (σ=10 was assumed known)

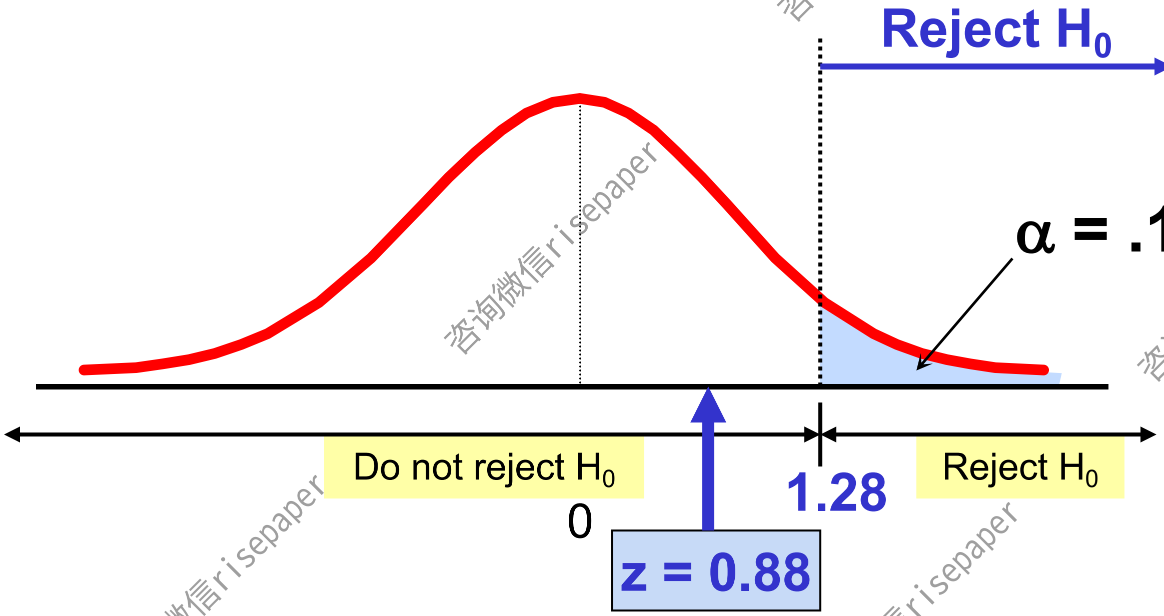
- Using the sample results,

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Example: Decision

(continued)

Reach a decision and interpret the result:



Do not reject H_0 since $z = 0.88 < 1.28$

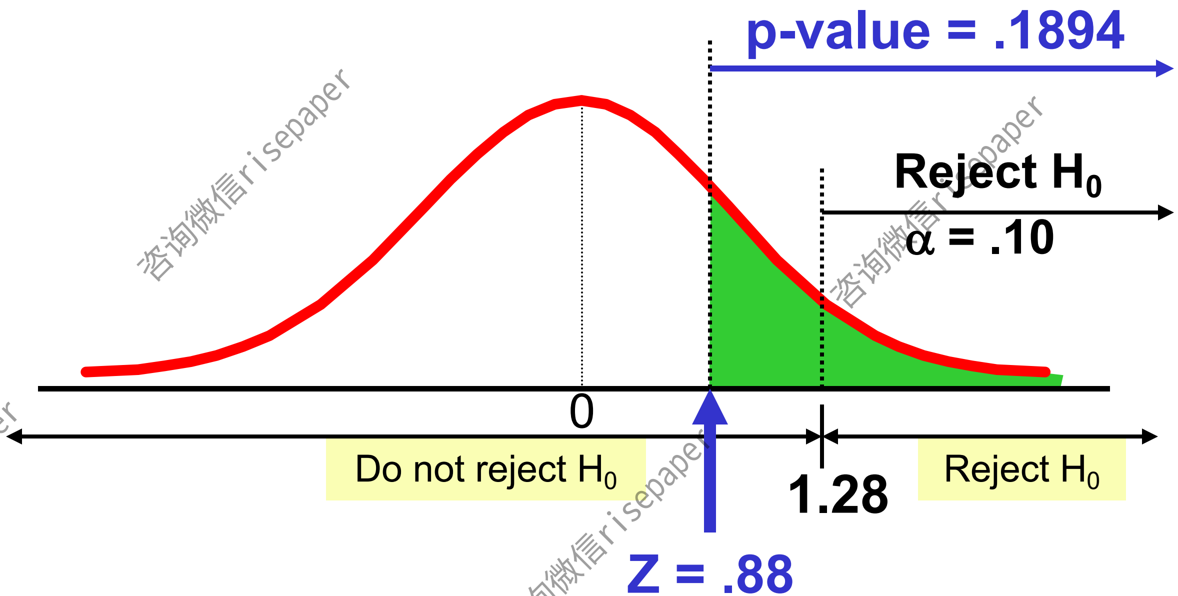
i.e.: there is not sufficient evidence that the mean bill is over $52

Example: p-Value Solution

(continued)

Calculate the p-value and compare to α

(assuming that μ = 52.0)



$$P(\bar{x} \geq 53.1 \mid \mu = 52.0)$$

$$= P\left(z \geq \frac{53.1 - 52.0}{10/\sqrt{64}}\right)$$

$$= P(z \geq 0.88) = 1 - .8106$$

Or =1-NORM.S.DIST(0.88,TRUE) $= .1894$

Do not reject H_0 since p-value = .1894 > α = .10

The Role α Plays in Hypothesis Testing

Changing α changes the critical z-score in the hypothesis test, which, in turn, changes the rejection region in the sampling distribution

TABLE 9.5 | Critical z-Scores for Various Alphas

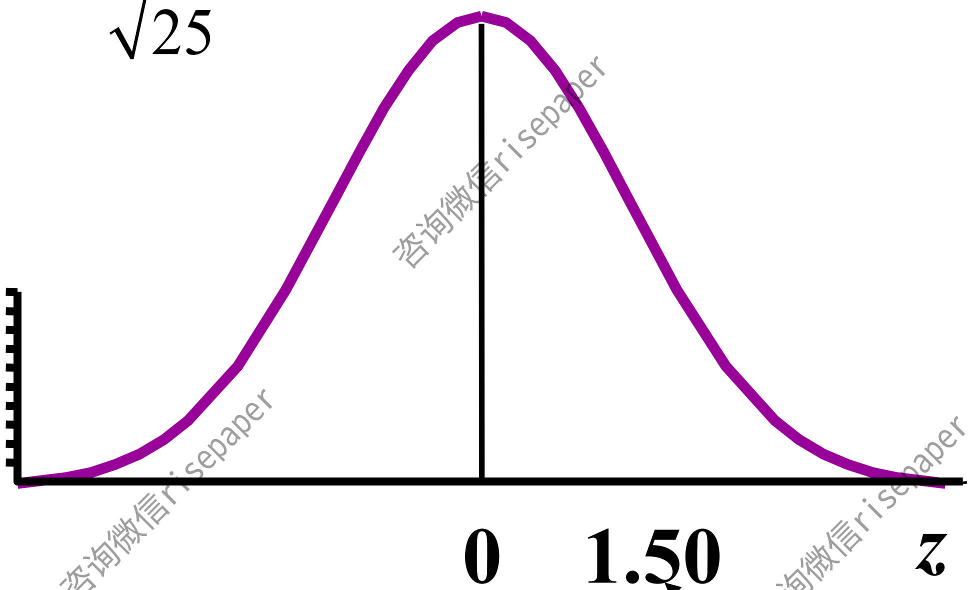
ALPHA (α)	TAIL	CRITICAL Z-SCORE
0.01	One	2.33
0.01	Two	2.575
0.02	One	2.05
0.02	Two	2.33
0.05	One	1.645
0.05	Two	1.96
0.10	One	1.28
0.10	Two	1.645

Two-Tailed *z* Test p-Value Example

Does an average box of cereal contain **368** grams of cereal? A random sample of **25** boxes showed $\bar{x}$ **= 372.5**. The company has specified σ to be **15** grams. Find the *p*-value. How does it compare to α = .05?

Two-Tailed *z* Test
p-Value Solution

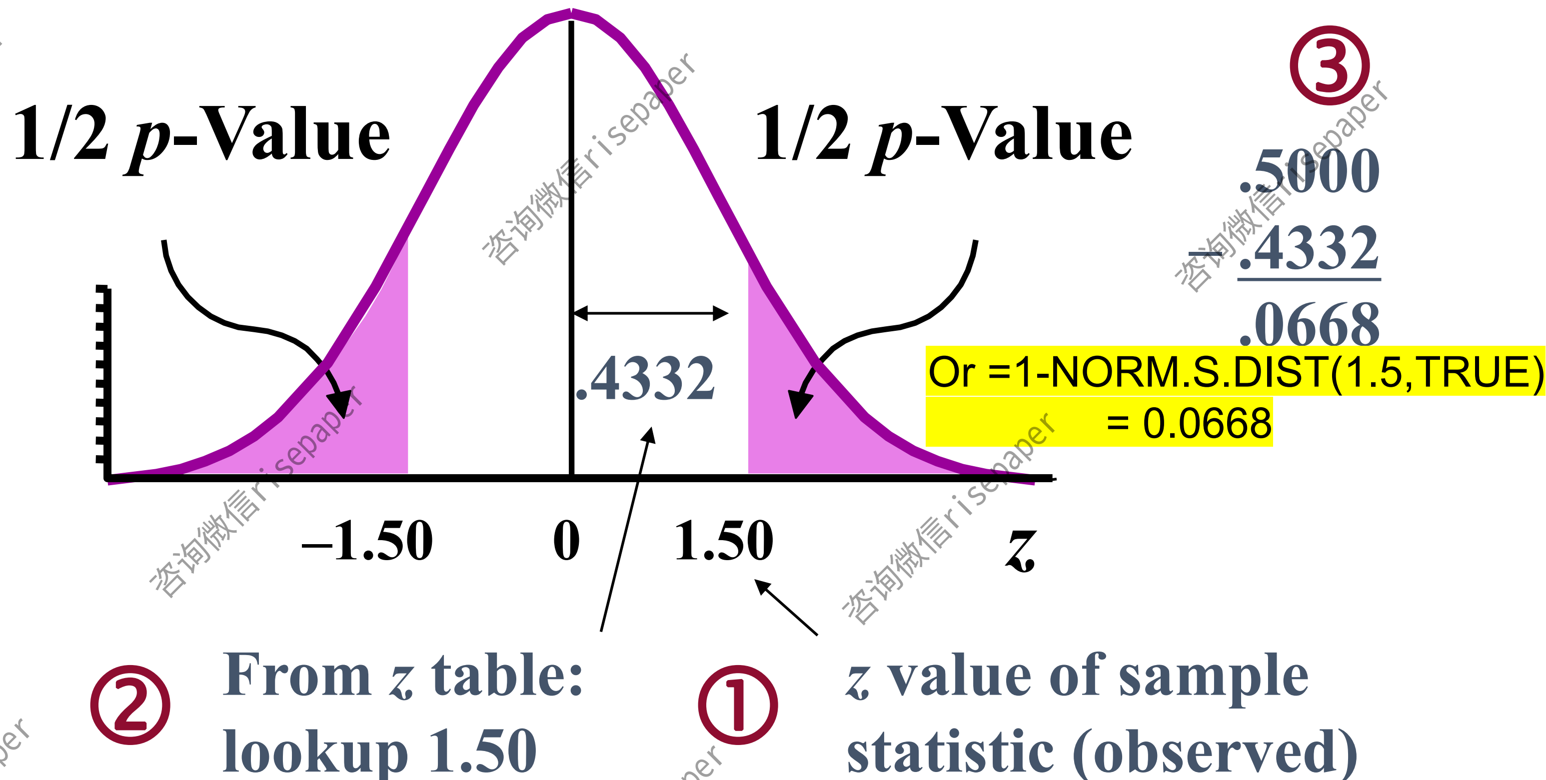
$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{372.5 - 368}{\frac{15}{\sqrt{25}}} = +1.50$$



① *z* value of sample statistic (observed)

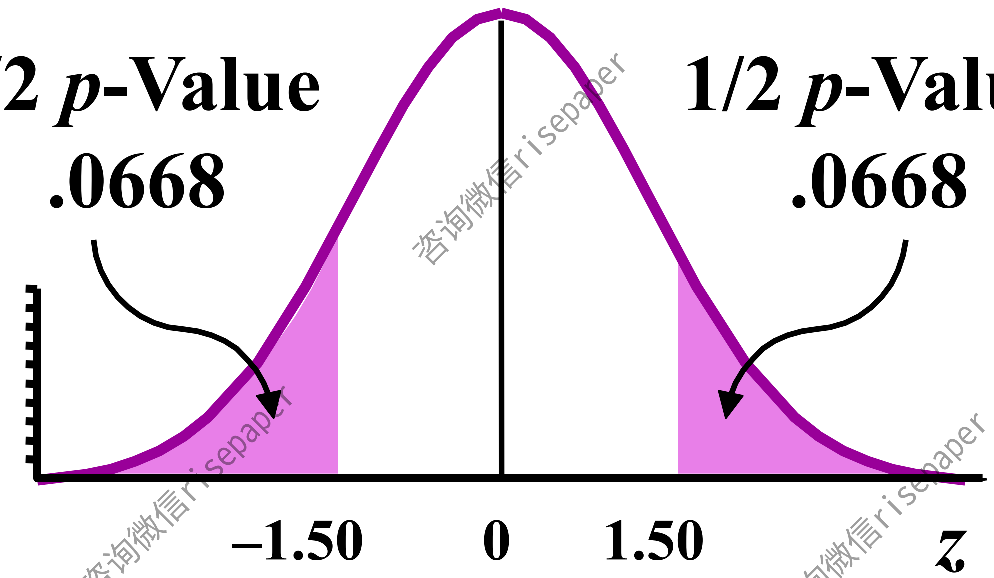
Two-Tailed *Z* Test
p-Value Solution

***p*-Value is $P(z \le -1.50 \text{ or } z \ge 1.50)$**



Two-Tailed *z* Test
p-Value Solution

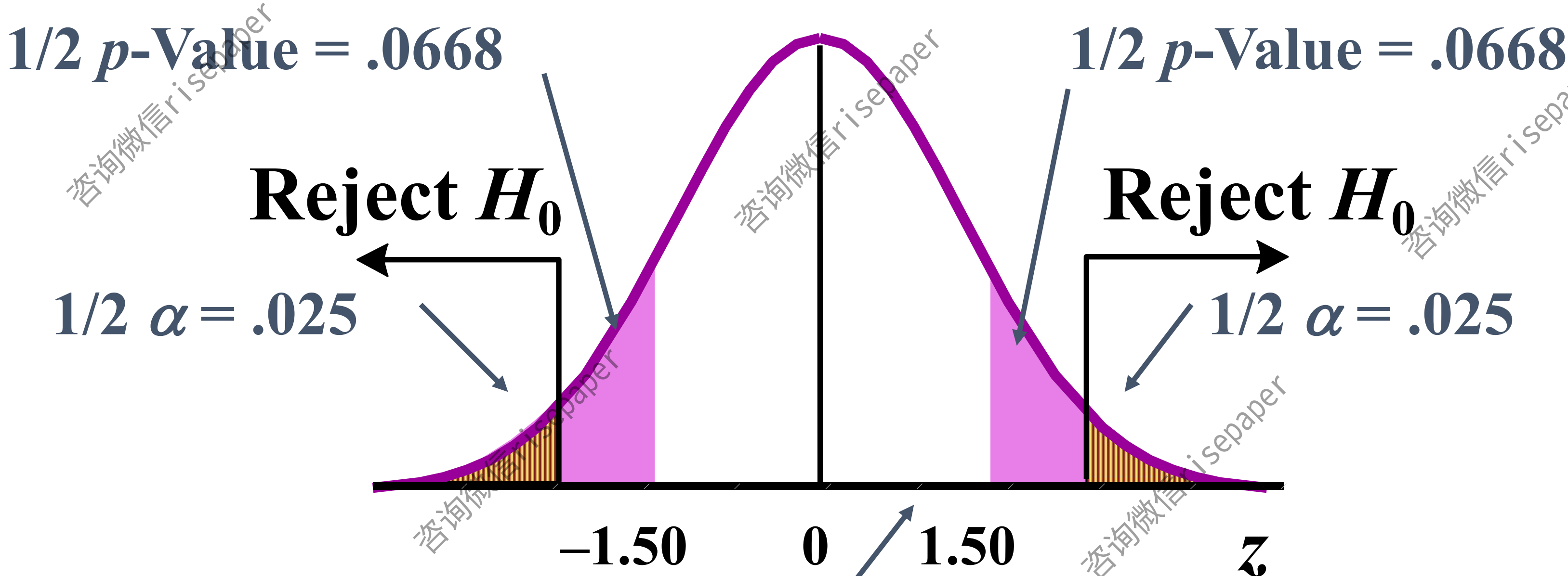
p-Value is $P(z \le -1.50 \text{ or } z \ge 1.50) = .1336$



Two-Tailed z Test
p-Value Solution

p-Value = .1336 ≥ α = .05

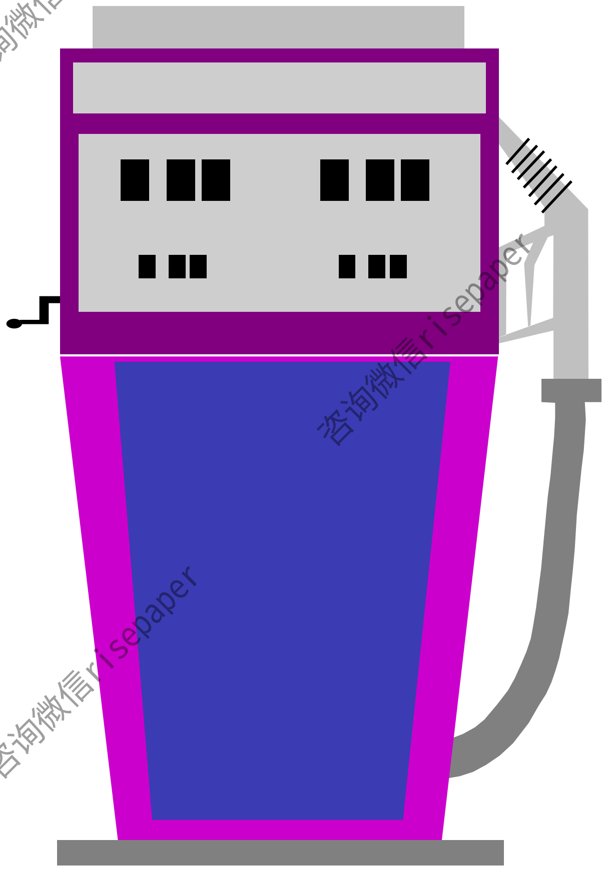
Do not reject H_0.



Test statistic is in ‘Do not reject’ region

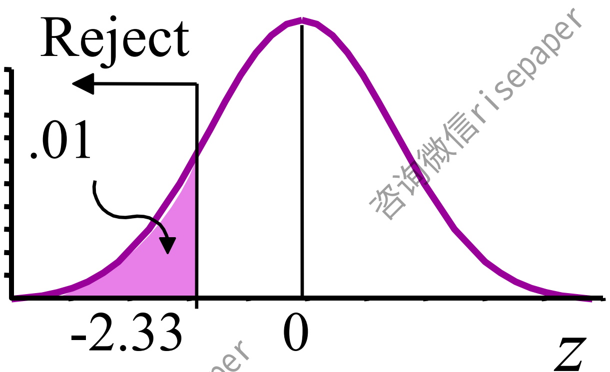
One-Tailed *z* Test Thinking Challenge

You're an analyst for Ford. You want to find out if the average miles per gallon of Escorts is at least 32 mpg. Similar models have a standard deviation of **3.8** mpg. You take a sample of **60** Escorts & compute a sample mean of **30.7** mpg. At the **.01** level of significance, is there evidence that the miles per gallon is **less than 32**?



One-Tailed *z* Test Solution*

- H_0: $\mu = 32$
- H_a: $\mu < 32$
- $\alpha =$ .01
- $n =$ 60
- Critical Value(s):



Test Statistic:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{30.7 - 32}{\frac{3.8}{\sqrt{60}}} = -2.65$$

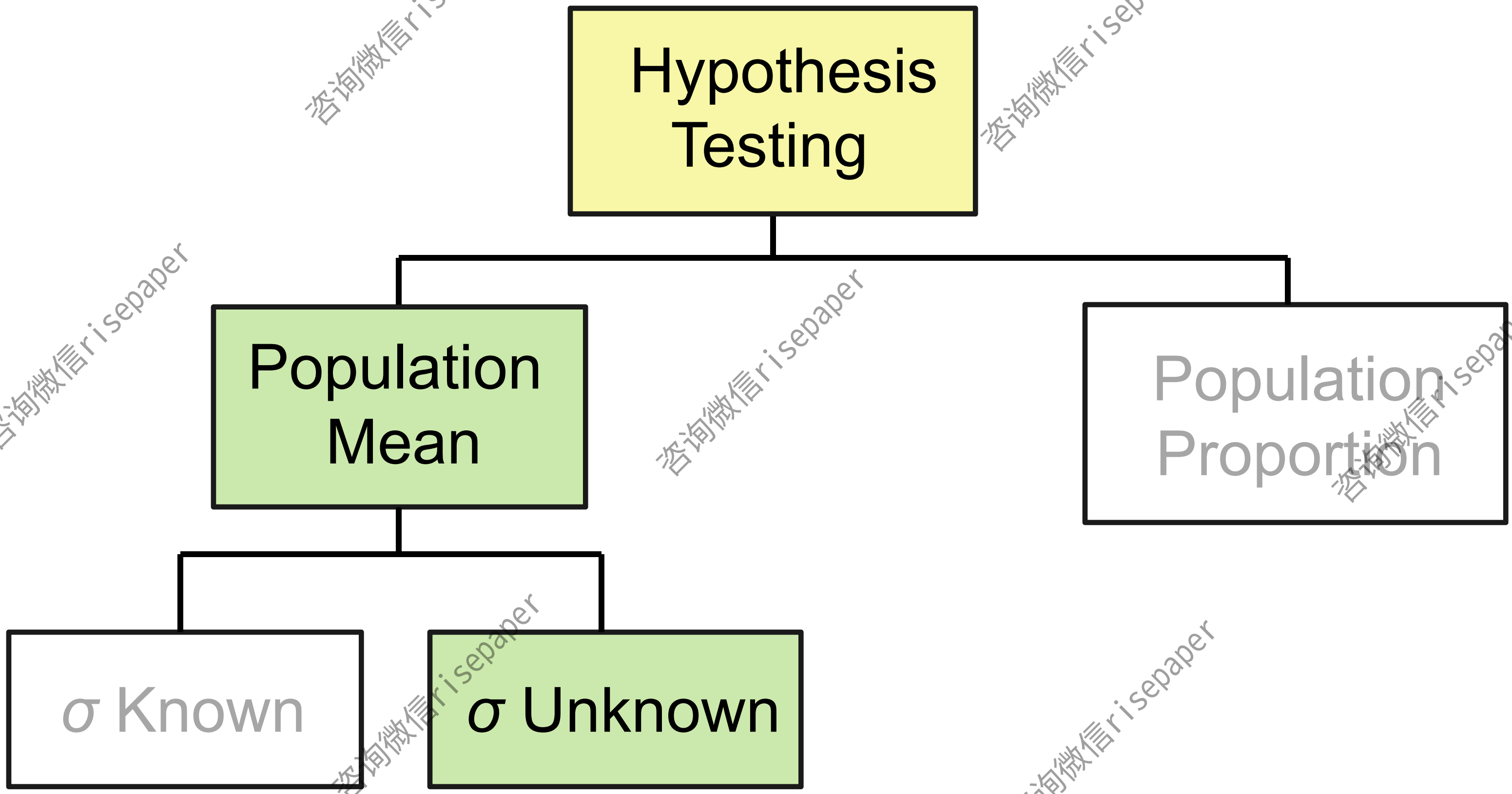
Decision:

Reject at $\alpha = .01$

Conclusion:

There is evidence average is less than 32

Hypothesis Testing for the Population Mean when σ is Unknown



Hypothesis Testing for the Population Mean when σ is Unknown

When the population standard deviation σ is unknown we **substitute the sample standard deviation**, s, in place of σ

- The sample standard deviation will be known once a sample has been collected

Use the Student's *t*-distribution for the test statistic rather than the normal distribution

- Assume that the population of interest follows the normal probability distribution

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: The average cost of a hotel room in Chicago is claimed to be $188 per night. A travel agent thinks it is lower now.

- A random sample of 25 hotels resulted in

$$\bar{x} = \$177.50 \text{ and } s = \$25.40$$

- Test the appropriate hypothesis using an $\alpha = 0.05$ level of significance (Assume the population distribution is normal)

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: (continued)

Step 1: Identify the null and alternative hypotheses

H_0: $\mu \geq \$188$	(status quo: average cost is not less than $188)
H_1: $\mu < \$188$	(the cost now is less than $188)

Step 2: Set a value for the significance level, α

- $\alpha = 0.05$ is specified for this test

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

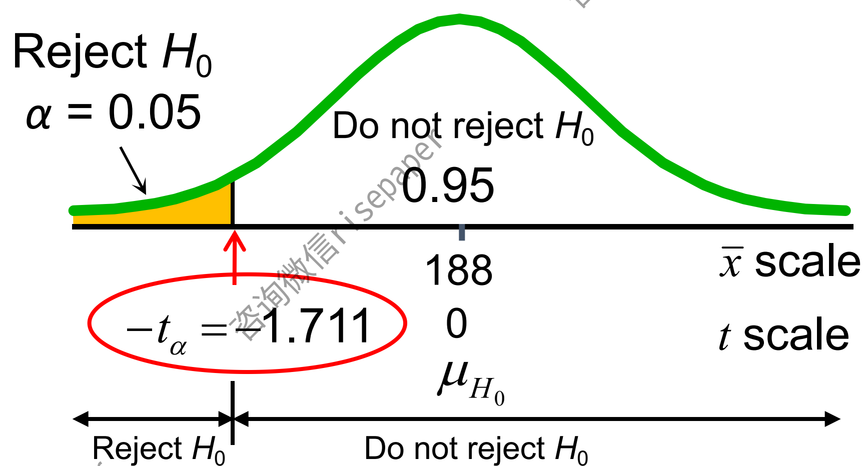
Example: (continued)

Step 3: Determine the appropriate **critical value**

- σ is unknown so use a *t*-score; *n* = 25 so use 24 degrees of freedom
- Since this is a one-tail test the entire area for α = 0.05 is placed on the left side (lower tail) of the sampling distribution:

The critical *t*-value can be found in Table in Appendix, or with Excel's **T.INV** function:
=T.INV(α, degrees_of_freedom)

This function returns the left-tail critical *t*-score, so use the absolute value of the result for an upper-tail test. Here,
= T.INV(0.05,24) = -1.711



An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: (continued)

Step 4: Calculate the appropriate test statistic

Formula for the *t*-test statistic for a hypothesis test for the population mean (when σ is unknown):

$$t_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}}$$

where:

$t_{\bar{x}}$ = The *t*-test statistic

$\bar{x}$ = The sample mean

μ_{H_0} = The mean of the sampling distribution, which is assumed to be true for the null hypothesis

s = The standard deviation of the sample

n = The sample size

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: (continued)

Step 4: Calculate the appropriate test statistic

$$t_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}} = \frac{177.50 - 188}{\frac{25.40}{\sqrt{25}}} = \frac{-10.5}{5.08} = -2.07$$

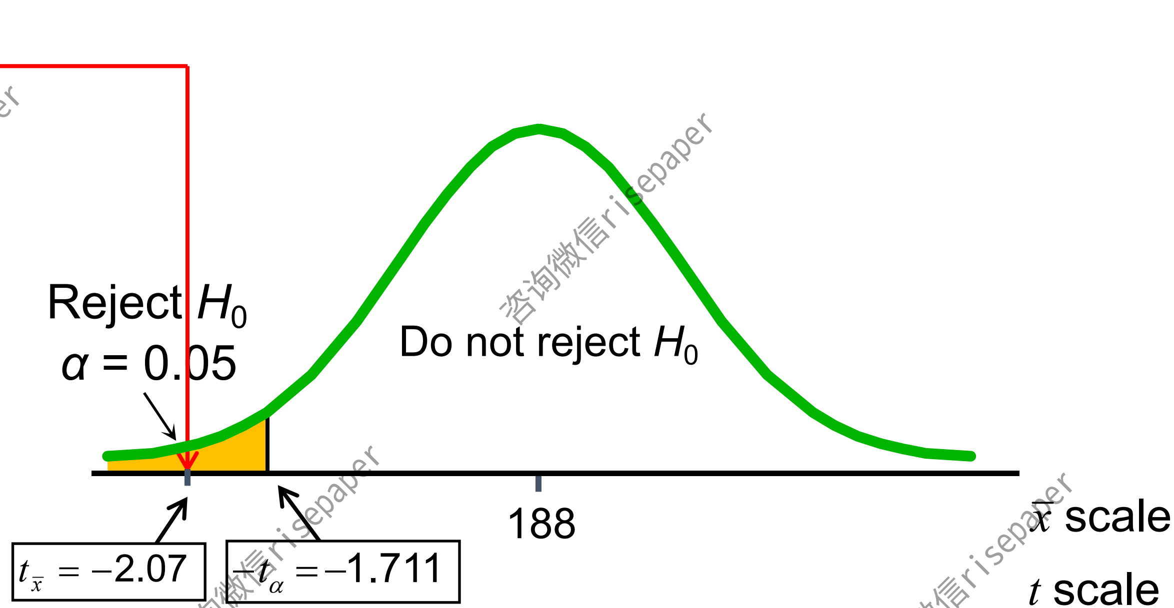
Step 5: Compare the t-test statistic $t_{\bar{x}}$ with the critical t-score t_{α}

- For a one-tail lower tail test, reject the null hypothesis if $t_{\bar{x}} < -t_{\alpha}$
- Here, -2.07 is less than -1.711, so **reject** $\boldsymbol{H_0}$
- (Illustrated on the next slide)

An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Step 5: (continued)

$t_{\bar{x}} = -2.07$ is less than $-t_\alpha = -1.645$, so **reject H_0**



An Example of a One-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: (continued)

Step 6: State the conclusion

- According to our sample evidence from 25 hotels, there is enough evidence to support that the average cost per night is now less than $188.

Estimating the p-value Using the Student's t-distribution

The p-value must be approximated from the t-table (from Table 5 in Appendix A)

- In the last example, the t-score was $t_{\bar{x}} = -2.07$
- We need to find the critical t-scores that bracket

$$|t_{\bar{x}}| = |-2.07| = 2.07$$

in the $(n - 1) = (25 - 1) = 24$ row in Table 5 from Appendix A

- The p-value will be between the two probabilities shown in the corresponding column headings

Estimating the *p*-value Using the Student's *t*-distribution

The *p*-value is between 0.025 and 0.010:

1 Tail	0.200	0.100	0.050	0.025	0.010	0.005
2 Tail	0.400	0.200	0.100	0.050	0.020	0.010
Conf Lev	0.600	0.800	0.900	0.950	0.980	0.990
df						
21	0.859	1.323	1.721	2.080	2.518	2.831
22	0.858	1.321	1.717	2.074	2.508	2.819
23	0.858	1.319	1.714	2.069	2.500	2.807
24	0.857	1.318	1.711	2.064	2.492	2.797
25	0.856	1.316	1.708	2.060	2.485	2.787
26	0.856	1.315	1.706	2.056	2.479	2.779

$t_{\bar{x}} = 2.07$

Estimating the *p*-value Using the Student's *t*-distribution

Even though this is not a precise *p*-value, using α = 0.05 we can still reject the null hypothesis

A precise *p*-value for a hypothesis test can be found using Excel's **T.DIST** function:

= T.DIST(*x*, degrees_of_freedom, cumulative)

When we set *x* to our test statistic $t_{\bar{x}} = -2.07$ and cumulative = TRUE, the function provides the area to the left of $t_{\bar{x}} = -2.07$ which is the desired *p*-value:

=T.DIST(-2.07, 24, TRUE) = 0.0247

An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: Claim -- the average person in the U.S. watches 34.5 hours of television per week

- Suppose viewing data is recorded for 10 people and the sample mean is found to be 39.6 hours and the sample standard deviation is 16.4 hours per week

- Test using $\alpha = 0.02$

The following slides show the steps to complete this hypothesis test

An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: (continued)

Step 1: Identify the null and alternative hypotheses

$H_0: \mu = 34.5$	(status quo: average time is 34.5 hours per week)
$H_1: \mu \neq 34.5$	(average viewing time is not equal to 34.5 hours per week)

Step 2: Set a value for the significance level, α

- Suppose that $\alpha = 0.02$ is chosen

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An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

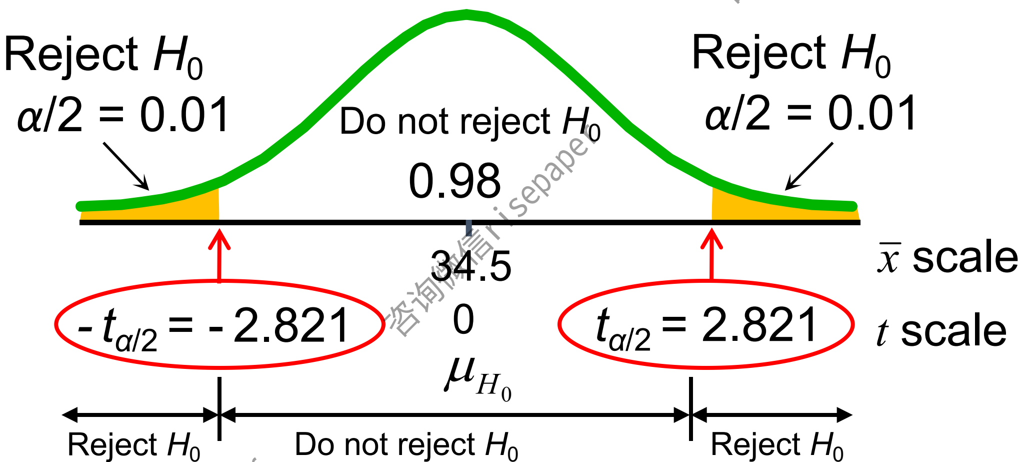
Example: (continued)

Step 3: Determine the appropriate **critical values**

- σ is unknown so use a *t*-score for α = 0.02 and $n - 1 = 9$ degrees of freedom
- Since this is a two-tail test, α = 0.02 is split equally into two tails:

The critical *t*-values can be found in Table 5 in Appendix A, or with Excel's **T.INV.2T** function:
=T.INV.2T(α, degrees_of_freedom)

Here,
=T.INV.2T(0.02,9) = 2.821



An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: (continued)

Step 4: Calculate the appropriate test statistic

$$t_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}} = \frac{39.6 - 34.5}{\frac{16.4}{\sqrt{10}}} = \frac{5.1}{5.19} = 0.98$$

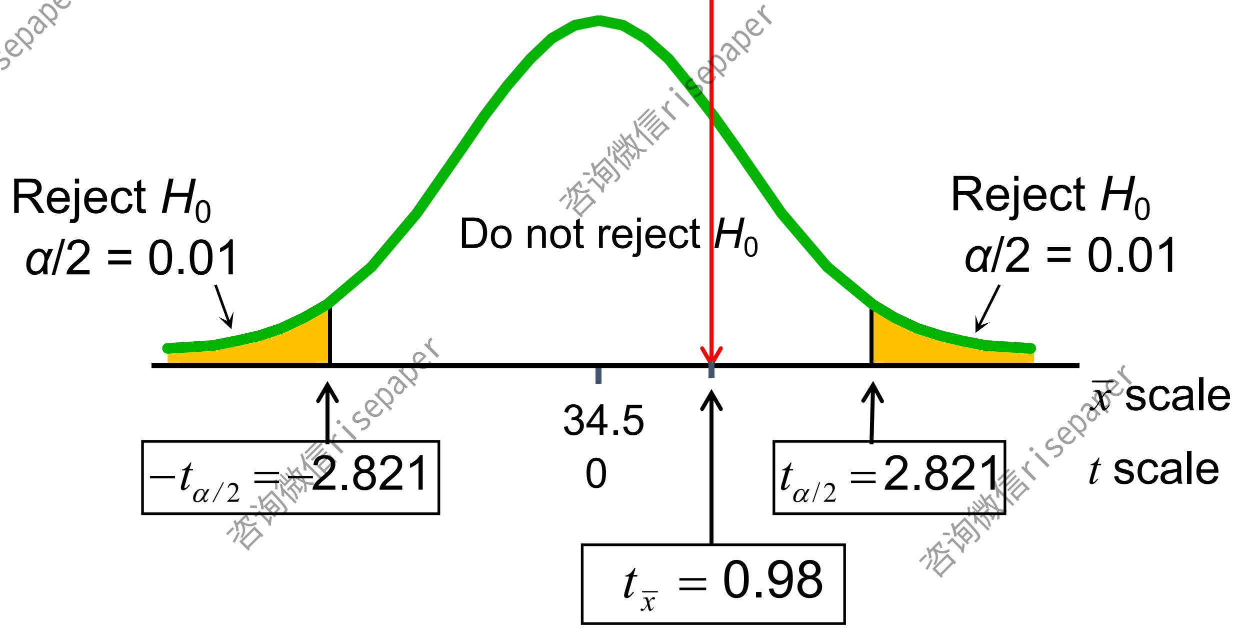
Step 5: Compare the *t*-test statistic $t_{\bar{x}}$ with the critical *t*-score $t_{\alpha/2}$

- For a two-tail test, reject the null hypothesis if $|t_{\bar{x}}| > |t_{\alpha/2}|$
- Here, 0.98 is not greater than 2.821, so **do not reject H_0**
- (Illustrated on the next slide)

An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Step 5: (continued)

$t_{\bar{x}} = 0.98$ is not greater than $t_{\alpha/2} = 2.821$, so **do not reject H_0**



An Example of a Two-Tail Hypothesis Test for the Population Mean (When σ Is Unknown)

Example: (continued)

Step 6: (Final Step) State the conclusion

- Our random sample of 10 viewers does not provide sufficient evidence to reject the null hypothesis, so we have no evidence to conclude that the average does not equal 34.5 hours per week.

Hypothesis Test for the Population Mean (when σ Is Unknown) by excel

We can also use Excel's **T.DIST.2T** function to determine the *p*-value for a two-tail test using the *t*-distribution:

=T.DIST.2T(*x*, degrees_of_freedom)

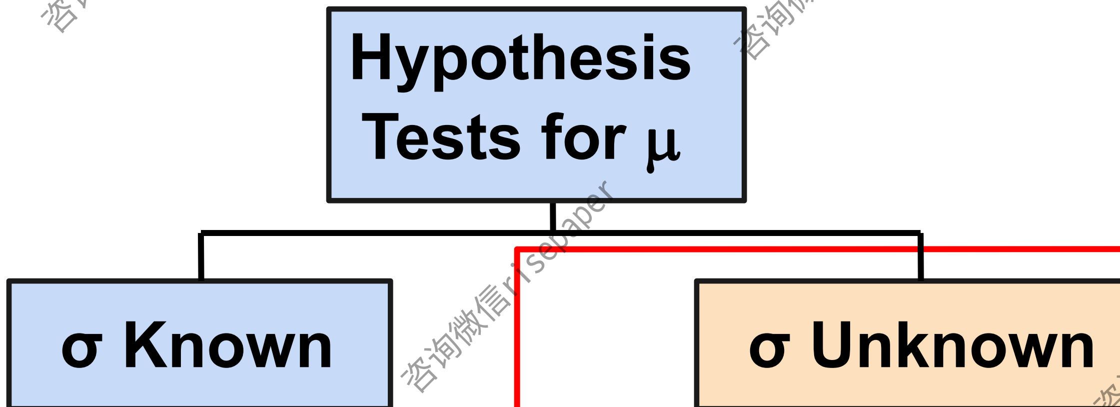
When we set $x = |t_{\bar{x}}|$, this function provides the area to the left of $t_{\bar{x}}$ plus the area to the right of $t_{\bar{x}}$, which is the desired *p*-value:

=T.DIST.2T(0.98, 9) = 0.3527

This *p*-value is slightly different than the *p*-value provided by PHStat on the last slide (0.3511) because $t_{\bar{x}}$ is rounded to 0.98 here

t Test of Hypothesis for the Mean (σ Unknown)

- Convert sample result ($\bar{x}$) to a t test statistic



Consider the test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$

(Assume the population is normal)

The decision rule is:

Reject H_0 if $t = \dfrac{\bar{x} - \mu_0}{\dfrac{s}{\sqrt{n}}} > t_{n-1,\alpha}$

t Test of Hypothesis for the Mean (σ Unknown)

(continued)

- For a two-tailed test:

Consider the test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

(Assume the population is normal, and the population variance is unknown)

The decision rule is:

Reject H_0 if $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} < -t_{n-1, \alpha/2}$ or if $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} > t_{n-1, \alpha/2}$

Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in New York is said to be \$168 per night. A random sample of 25 hotels resulted in $\bar{x}$ = \$172.50 and s = \$15.40. Test at the α = 0.05 level.

(Assume the population distribution is normal)

H_0: $\mu = 168$

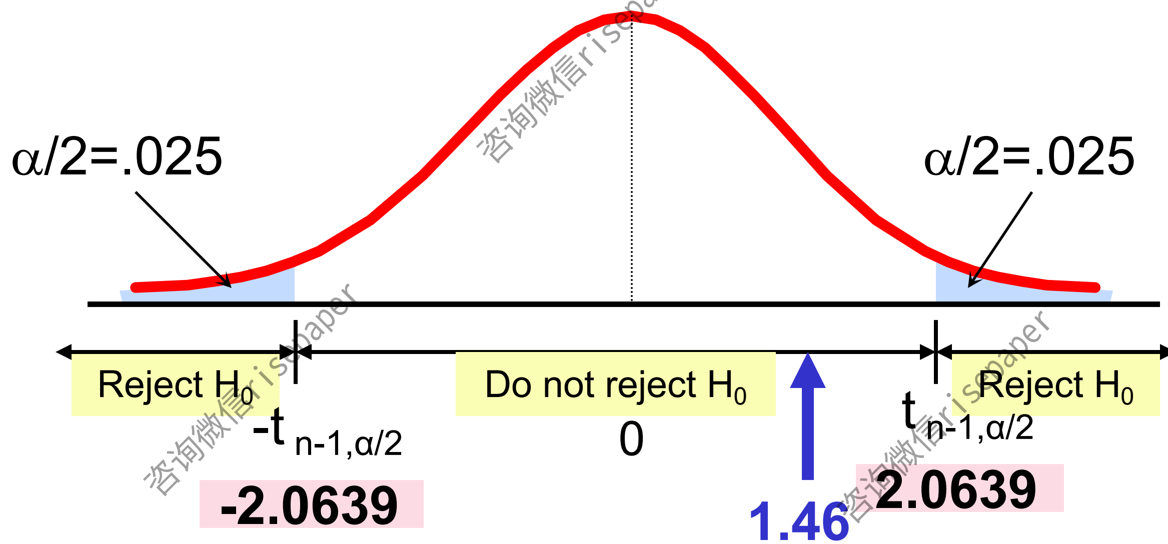
H_1: $\mu \neq 168$

Example Solution: Two-Tail Test

H_0: $\mu = 168$
H_1: $\mu \neq 168$

- $\alpha = 0.05$
- $n = 25$
- σ is unknown, so use a **t statistic**
- Critical Value:

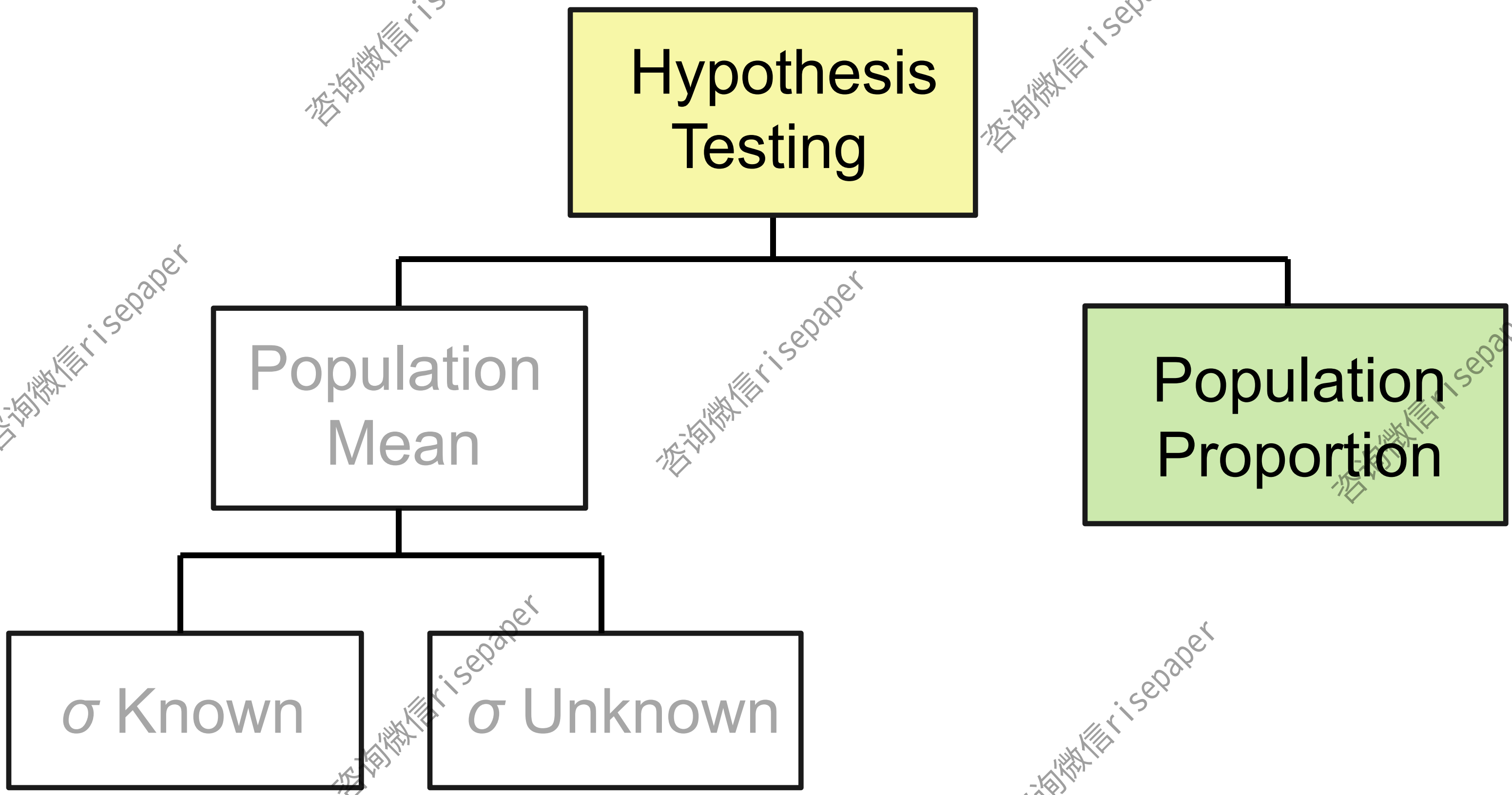
$t_{24\,,\,.025} = \pm\, 2.0639$



$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0: not sufficient evidence that true mean cost is different than $168

Hypothesis Testing for the Proportion of a Population



Tests of the Population Proportion

- Involves categorical variables
- Two possible outcomes
  - "Success" (a certain characteristic is present)
  - "Failure" (the characteristic is not present)
- Fraction or proportion of the population in the "success" category is denoted by P
- Assume sample size is large

Proportions

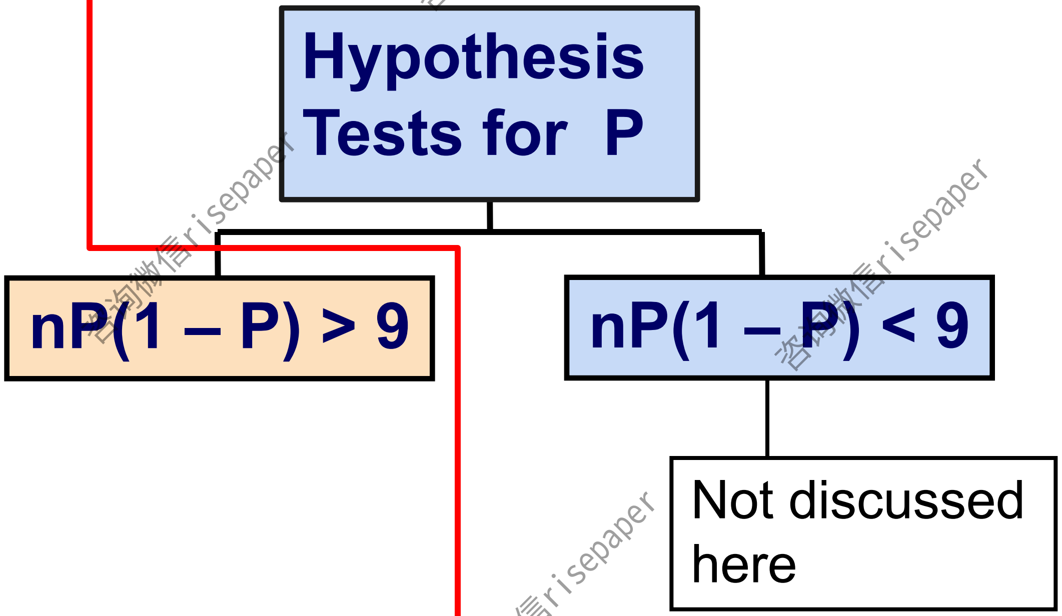
(continued)

- Sample proportion in the success category is denoted by $\hat{p}$
  - $$\hat{p} = \frac{x}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$$
- When $nP(1 - P) > 9$, $\hat{p}$ can be approximated by a normal distribution with mean and standard deviation
  - $$\mu_{\hat{p}} = P \qquad \sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}}$$

Hypothesis Tests for Proportions

- The sampling distribution of $\hat{p}$ is approximately normal, so the test statistic is a z value:

$$z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}}$$



Hypothesis Testing for the Proportion of a Population

Used to test a hypothesis about a population proportion

- Proportion data follow the binomial distribution, which can be approximated by the normal distribution if:

$$np \geq 5 \quad \text{and} \quad n(1-p) \geq 5$$

where:

p = The probability of a success in the population

n = The sample size

Hypothesis Testing for the Proportion of a Population

Formula for the Sample Proportion:

$$\bar{p} = \frac{x}{n}$$

where:

x = The number of observations of interest in the sample (successes)

n = The sample size

Hypothesis Testing for the Proportion of a Population

Formula for the z-test Statistic for a Hypothesis Test for the Proportion:

$$z_p = \frac{\bar{p} - p_{H_0}}{\sqrt{\frac{p_{H_0}(1 - p_{H_0})}{n}}}$$

where:

z_p = The z-test statistic for the proportion

$\bar{p}$ = The sample proportion

p_{H_0} = The population proportion, which is assumed to be true in the null hypothesis

n = The sample size

An Example of a One-Tail Hypothesis Test for the Proportion

- **Example:** The proportion of cell phone users with 4G contracts last year was $p = 0.62$. A Verizon executive thinks the proportion has increased this year.

  - Suppose that from a random sample of 350 users, 238 have 4G contracts

  - Test using $\alpha = 0.05$

An Example of a One-Tail Hypothesis Test for the Proportion

Example: (continued)

Step 1: Identify the null and alternative hypotheses

H_0: $p \leq 0.62$	(status quo: the proportion is not greater than 0.62)
H_1: $p > 0.62$	(the proportion has increased from 0.62)

Step 2: Set a value for the significance level, α

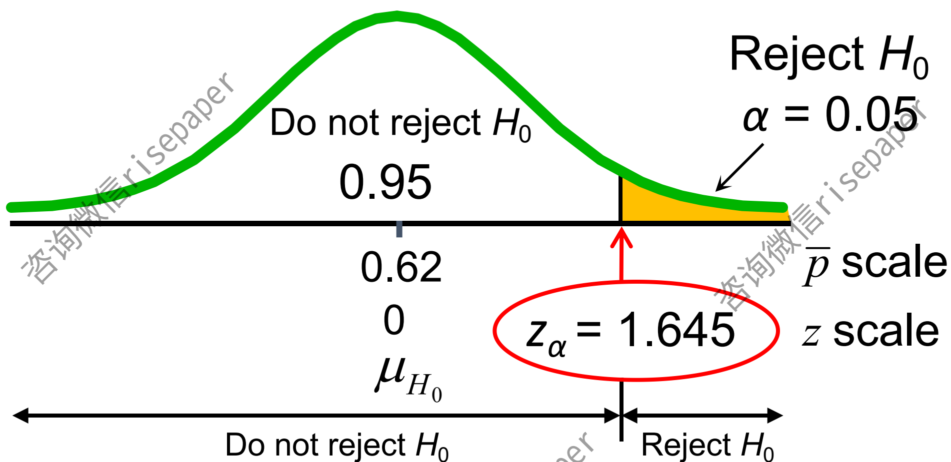
- $\alpha = 0.05$ has been chosen

An Example of a One-Tail Hypothesis Test for the Proportion

Example: (continued)

Step 3: Determine the appropriate **critical value**

- The binomial distribution can be approximated with the normal distribution, so use a *z*-score: for $\alpha = 0.05$ the *z*-score is 1.645
- Since this is a one-tail test the entire area for $\alpha = 0.05$ is placed on the right side (upper tail) of the sampling distribution:



An Example of a One-Tail Hypothesis Test for the Proportion

Example: (continued)

Step 4: Calculate the appropriate test statistic

Find the sample proportion and the *z*-test statistic for a hypothesis test for the population proportion:

$$\bar{p} = \frac{x}{n} = \frac{238}{350} = 0.68$$

$$z_p = \frac{\bar{p} - p_{H_0}}{\sqrt{\frac{p_{H_0}(1 - p_{H_0})}{n}}} = \frac{0.68 - 0.62}{\sqrt{\frac{0.62(1 - 0.62)}{350}}} = \frac{0.06}{0.026} = 2.31$$

An Example of a One-Tail Hypothesis Test for the Proportion

Example: (continued)

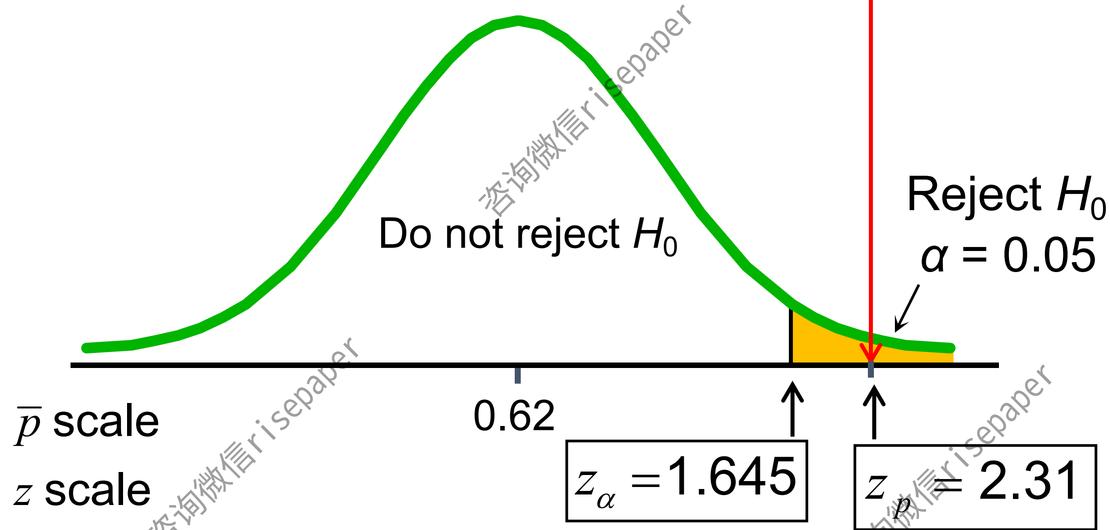
Step 5: Compare the *z*-test statistic z_p with the critical *z*-score z_α

- For a one-tail upper tail test, reject the null hypothesis if $z_p > z_\alpha$
- Here, 2.31 is greater than 1.645, so **reject H_0**
- (Illustrated on the next slide)

An Example of a One-Tail Hypothesis Test for the Proportion

Step 5: (continued)

$z_p = 2.31$ is greater than $z_\alpha = 1.645$, so **reject H_0**



An Example of a One-Tail Hypothesis Test for the Proportion

Example: (continued)

Step 6: State the conclusion

We can conclude at the 0.05 level of significance that the proportion of people with 4G contracts has increased above 62%.

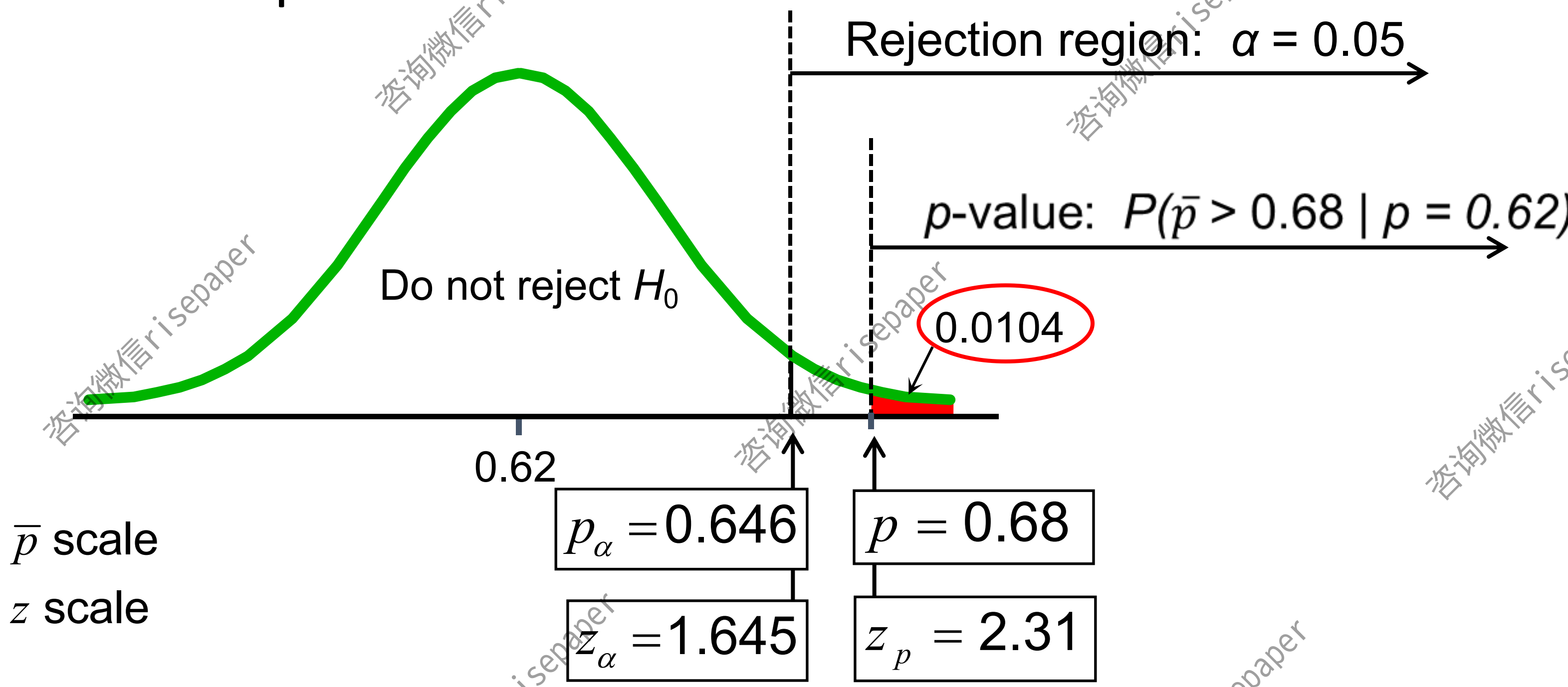
The p-value Approach to Hypothesis Testing for the Proportion

The *p*-value procedure for the hypothesis test for proportions is identical to the procedure for the mean

- Shows the probability of obtaining a sample result at least as unusual as the one observed, given that the null hypothesis is true

The p-value Approach to Hypothesis Testing for the Proportion

Prior example:



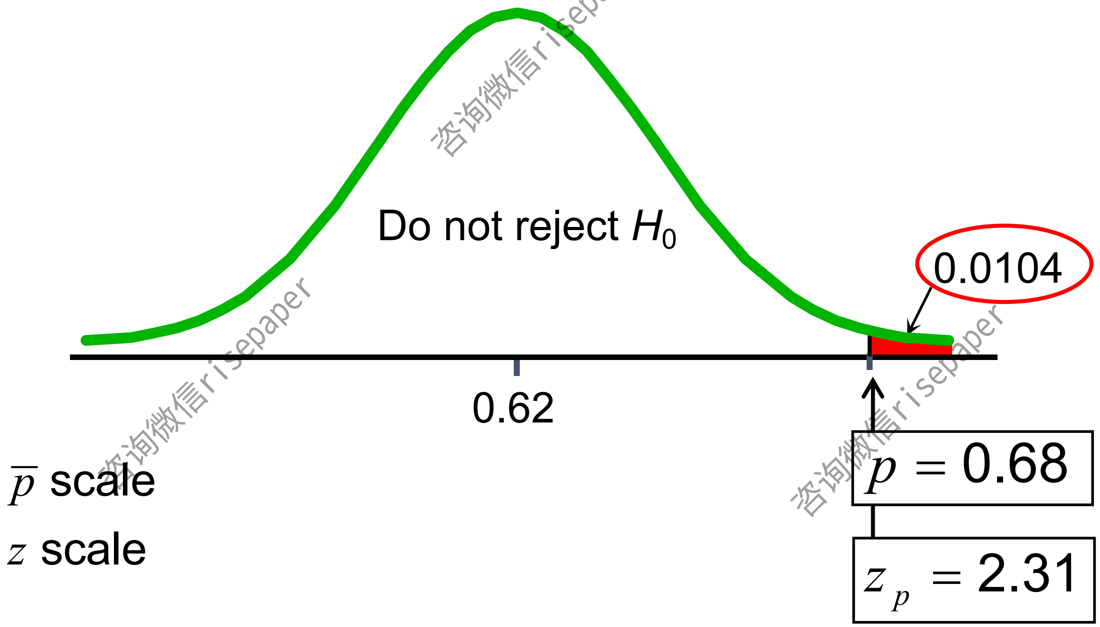
$$P(\bar{p} > 0.68) = P(z_p > 2.31) = 1 - 0.9896 = 0.0104$$

Since the p-value = 0.0104 < α = 0.05, we reject H_0

The p-value Approach to Hypothesis Testing for the Proportion

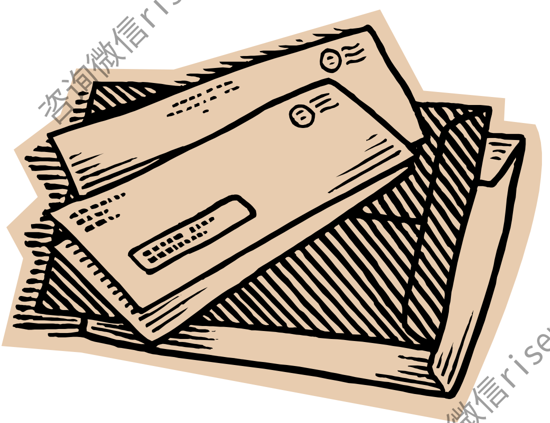
Excel's **NORM.S.DIST** function can be used to determine the *p*-value for this example:

= 1.0 - NORM.S.DIST(2.31, TRUE) = 1.0 - 0.9896 = 0.0104



Example: Z Test for Proportion

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the $\alpha = .05$ significance level.



Check:

Our approximation for P is

$\hat{p} = 25/500 = .05$

$nP(1 - P) = (500)(.05)(.95)$
$= 23.75 > 9$ ✓

Z Test for Proportion: Solution

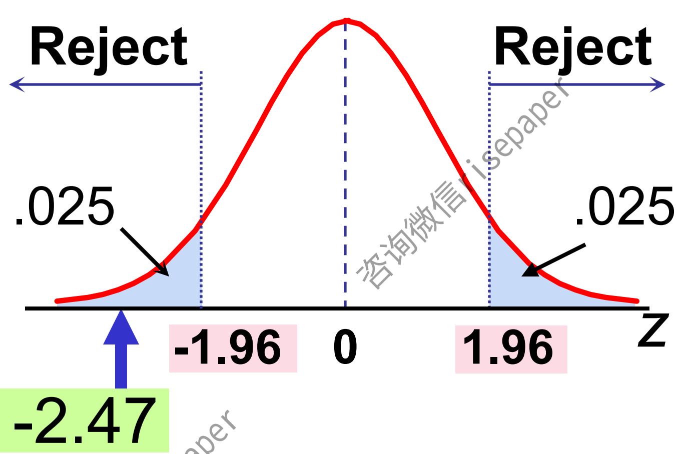
H_0: P = .08

H_1: P ≠ .08

α = .05

n = 500, $\hat{p}$ = .05

Critical Values: ± 1.96



Test Statistic:

$$z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{.05 - .08}{\sqrt{\frac{.08(1-.08)}{500}}} = -2.47$$

Decision:

Reject H_0 at α = .05

Conclusion:

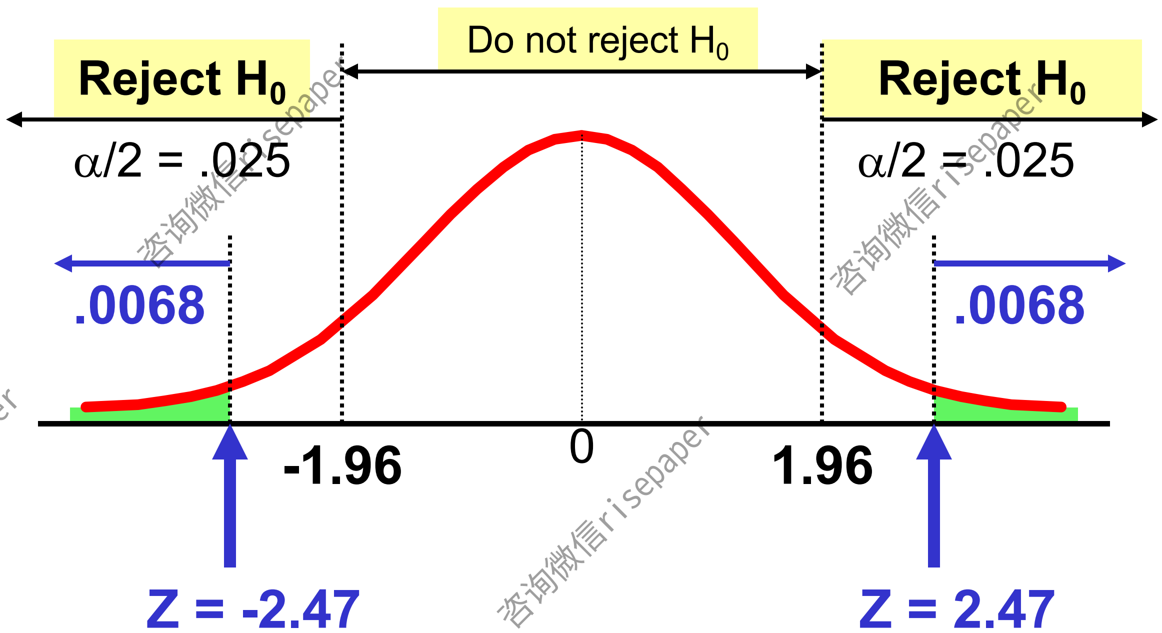
There is sufficient evidence to reject the company's claim of 8% response rate.

p-Value Solution

(continued)

Calculate the p-value and compare to α

(For a two sided test the p-value is always two sided)



p-value = .0136:

$$P(Z \le -2.47) + P(Z \ge 2.47)$$

$$= 2(.0068) = 0.0136$$

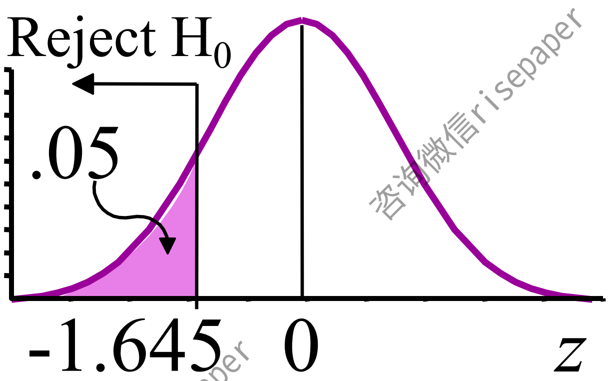
Reject H_0 since p-value = .0136 < α = .05

One-Proportion *z* Test Example

The present packaging system produces **10%** defective cereal boxes. Using a new system, a random sample of **200** boxes had **11** defects. Does the new system produce proportionately **fewer** defects? Test at the **.05** level of significance.

One-Proportion z Test Solution

- H_0: $p = .10$
- H_a: $p < .10$
- α = .05
- n = 200
- Critical Value(s):



Test Statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{\frac{11}{200} - .10}{\sqrt{\frac{.10(.90)}{200}}} = -2.12$$

Decision:

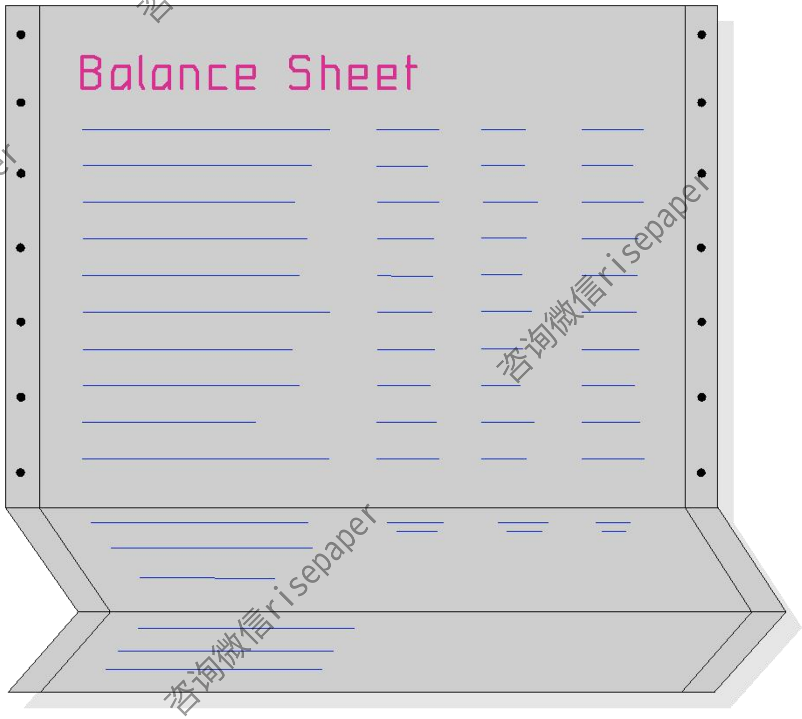
Reject at $\alpha = .05$

Conclusion:

There is evidence new system < 10% defective

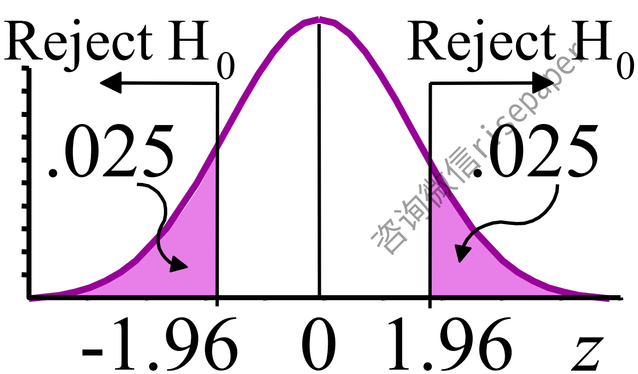
One-Proportion *z* Test Thinking Challenge

You're an accounting manager. A year-end audit showed **4%** of transactions had errors. You implement new procedures. A random sample of **500** transactions had **25** errors. Has the **proportion** of incorrect transactions **changed** at the **.05** level of significance?



One-Proportion z Test Solution*

- H_0: $p = .04$
- H_a: $p \neq .04$
- $\alpha =$ .05
- $n =$ 500
- Critical Value(s):



Test Statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{\frac{25}{500} - .04}{\sqrt{\frac{.04(.96)}{500}}} = 1.14$$

Decision:

Do not reject at $\alpha = .05$

Conclusion:

There is evidence proportion is not 4%

Type II Errors

The probability of a **Type II error** is known as β

- β is the probability of failing to reject a false H_0

Once we set the value for α, the value for β becomes fixed

•The values for α and β are inversely related for a constant sample size

•The only way to lower the probability of both errors at the same time is to increase the sample size

Calculating the Probability of Type II Errors Occurring

To calculate β we assume that the null hypothesis is not true and should be rightfully rejected

- The value of β depends on the actual value of μ, the population mean that is true if H_0 is false
- Even though we don't know μ, we can specify a value for μ and calculate the corresponding value of β

Calculating the Probability of Type II Errors Occurring

As the true population mean moves away from the hypothesized mean, the power of the test increases

- This makes it easier to correctly reject the null hypothesis

When the true population mean is very close to the hypothesized mean, the power of the test becomes very small

- This makes it difficult for the hypothesis test to accurately reject the null hypothesis

Decision	Actual Situation			
	Hypothesis Testing		Legal System	
	H0 True	H0 False	Innocence	Not innocence
Do Not Reject H_0	**No Error** $(1-\alpha)$	**Type II Error** (β)	No Error (not guilty, found not guilty) $(1-\alpha)$	Type II Error (guilty, found not guilty) (β)
Reject H_0	**Type I Error** (α)	**No Error** $(1-\beta)$	Type I Error (Not guilty, found guilty) (α)	No Error (guilty, found guilty) $(1-\beta)$

- Type I and Type II errors cannot happen at the same time

1. Type I error can only occur if H_0 is true
2. Type II error can only occur if H_0 is false
3. There is a tradeoff between type I and II errors. If the probability of type I error (α) increased, then the probability of type II error (β) declines.
4. When the difference between the hypothesized parameter and the actual true value is small, the probability of type two error (the non-rejection region) is larger.
5. Increasing the sample size, n, for a given level of α, reduces β

Review of Hypo. Testing

What is Hypothesis Testing?

Probability of making erroneous conclusions

- Type I – only when Null Hypo is true
- Type II – only when Null Hypo is false

Two Approaches

- The Rejection or Critical Value Approach
- The P-value Approach (we calculate the observed level of significance)

Test Statistics

- Z- distribution if Population Std. Dev. is Know
- t-distribution if the Population Std. Dev. is unknown

Test of Hypothesis for the Mean

σ known

The test statistic is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

σ Unknown

The test statistic is:

$$t_{n-1} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

Rejection Region or Critical Value Approach:

The given level of significance = α

H_0: μ = 12
H_1: μ ≠ 12

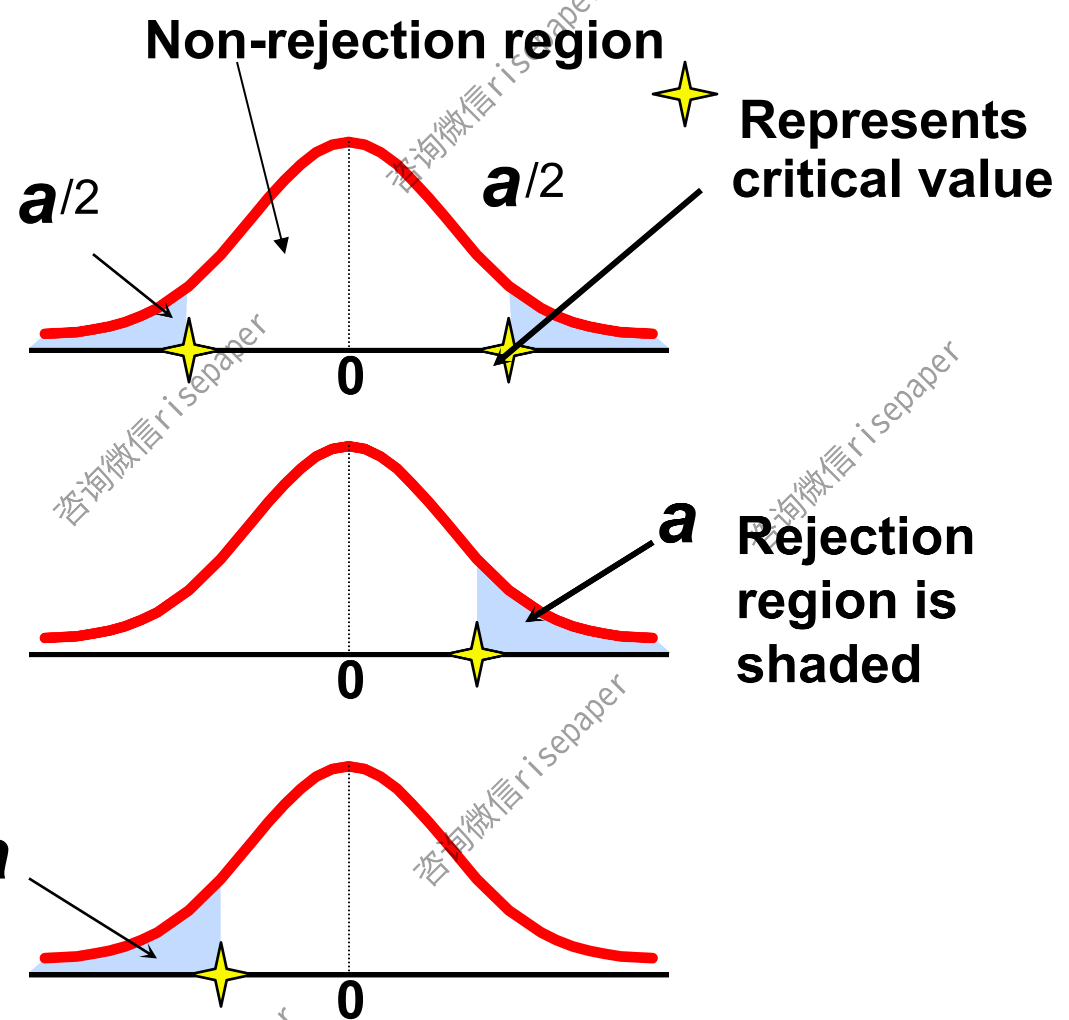
Two-tail test

H_0: μ ≤ 12
H_1: μ > 12

Upper-tail test

H_0: μ ≥ 12
H_1: μ < 12

Lower-tail test



P-Value Approach –

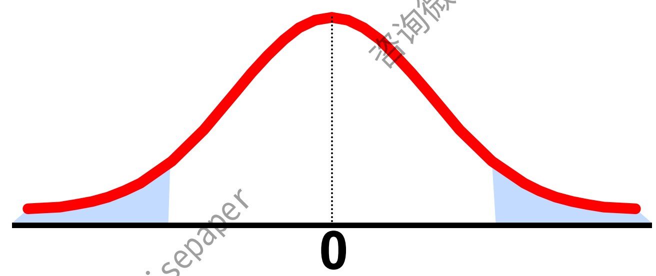
- P-value=Max. Probability of (Type I Error), calculated from the sample.

Given the sample information what is the size of the blue areas? (The observed level of significance)

H_0: μ = 12

H_1: μ ≠ 12

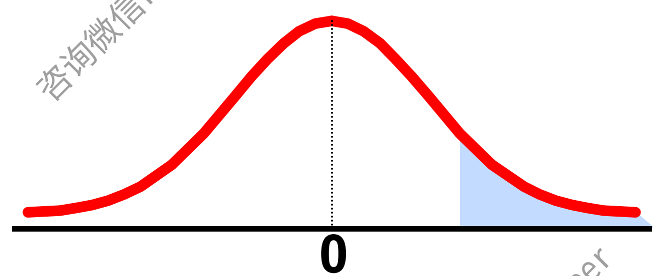
Two-tail test



H_0: μ ≤ 12

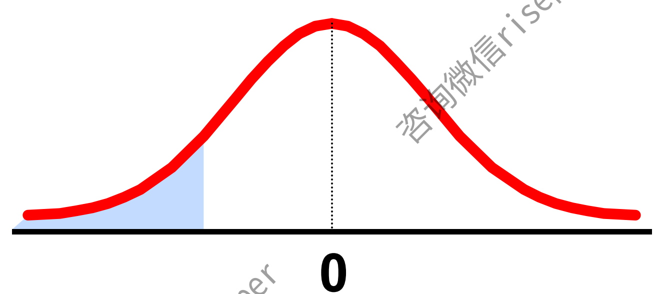
H_1: μ > 12

Upper-tail test



H_0: μ ≥ 12

H_1: μ < 12



Connection to Confidence Intervals

While the confidence interval estimation and hypothesis testing serve different purposes, they are based on same concept and conclusions reached by two methods are consistent for a two-tail test.

In CI method we estimate an interval for the population mean with a degree of confidence. If the estimated interval contains the hypothesized value under the hypothesis testing, then this is equivalent of not rejecting the null hypothesis. For example: for the beer sample with mean 5.20, the confidence interval is:

$$P(4.61 \leq \mu \leq 5.78) = 95\%$$

Since this interval contains the Hypothesized mean (\$4.90), we do not (did not) reject the null hypothesis at $\alpha = .05$

Did not reject and within the interval, thus consistent results.