W4415: Problem Set #3

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1. Consider the extensive-form split or steal game presented in class:

   https://www.youtube.com/watch?v=S0qjK3TWZE8

   Let X be the total amount they are playing over, so that if they they both split, the pay off would be $(X/2, X/2)$.

   (a) Consider first the one-shot split or steal game as it is normally played where players only care about monetary payoffs. What is/are the pure strategy Nash equilibrium of this game?

   (b) Suppose one of the players (call him the proposer) promises to steal. Suppose that player has a cost ψ that he has to pay if he breaks his promise. What is/are the pure strategy Nash equilibrium of the game. Would the proposer ever promise to split in this game without other additions to the model? Why or why not?

   (c) How does the previous question change if each player feels spite cost $\gamma > 0$ if and only if they wind up with a lower monetary payoff than the other person?

   (d) Now consider the two period extensive form of the game as in the video, where one of the players offers to split the money after the show. Find all of the subgame perfect nash equilibrium(s) assuming the only thing each player cares about is monetary payoffs (i.e., the players do not consider ψ or γ).

   (e) How does this change if you add back in ψ and γ for small ψ?

   (f) (For (f) and (g), keep γ but drop ψ.) Now suppose we are modeling honesty as follows: suppose that there is some small probability ϵ that the proposer is super honest, and never lies. That is, between the first stage and second stage of the game, let there be a move by Nature. With probability ϵ the proposer has no choice but to make the move (split or steal) that he promised before the game. With probability $1-\epsilon$ the game proceeds as normal.

   Can the proposer use this fact about himself to make a promise in the one-shot game to make sure the other player will choose to split? Why or why not?

   (g) How high would ϵ have to be in the two stage extensive form game to make the responder want to play split in equilibrium if the proposer promises to always split in the 2nd stage?

   (h) How else might you model what actually transpired in the video?

2. There are n firms: 1, 2, ..., n. Each firm has zero marginal cost. In the given order firms choose their production level, knowing what the previous firms have chosen. That is, Firm 1 chooses $q_1 \geq 0$ first; Firm 2 observes q_1 and chooses $q_2 \geq 0$; Firm 3 observes q_1 and q_2 and chooses $q_3 \geq 0$, and so on. They sell their good at market price $P = 1 - (q_1 + \cdots + q_n)$. Each firm i gets the payoff of $q_i P$. Compute a Subgame Perfect Nash equilibrium of this game using backward induction for the two following two cases:

   (a) for $n = 3$,

   (b) for arbitrary n.

3. Consider a model of (Bertrand) price competition. We have two duopoly stores that set prices simulatenously each week in a market whose demand curve is given by

$$Q = 6 - p$$

   where p is the lower of the two prices (and the lower priced store meets all demand). If the two stores post the same price p, each gets half the market; that is each gets $(6 - p)/2$. Suppose that prices can only be quoted in integer dollar amounts and the costs of production are zero. Suppose the two stores compete repeatedly, every week over the period of two years.

   (a) Sketch the extensive form of this game. (No need to draw all the iterations of course, just enough to illustrate). Describe the set of strategies for each store in the game.

   (b) Describe the Nash Equilibrium of the sub-game.

   (c) Explain how the sub-game perfect equilibrium of the full game allows both firms to collude at just about any price.

4. Ann and Bob are the heads of marketing and R&D departments in a company, respectively. Ann determines the level $a \in [0, 1]$ of advertisement and Bob determines the quality $b \in [0, 1]$ of the product. The profit of the firm is $\pi(a, b) = ab$. Ann and Bob have shares $s_A > 0$ and $s_B > 0$ in the profit, repectively. The cost of advertisement is a^γ for Ann, and the cost of product development is b^γ for Bob where $\gamma > 2$. Therefore, the payoffs of Ann and Bob are $s_A ab - a^\gamma$ and $s_B ab - b^\gamma$, respectively. They take actions simultaneously. Find all the Nash equilibria in pure strategies.

5. (Economic Naturalist) Describe a situation from your daily life, from the news, from a movie or television show, etc. that can be modeled using one of the concepts presented in class to date. Write down a model that describes the situation. Find numbers to calibrate your model. Solve the model for useful implications (policy responses, popular misconceptions, etc.). You will be judged on the interestingness/novelty/usefulness of your model and its implications, as well as the technical correctness of your model.