ECON701

APPLIED MICROECONOMICS

Assignment 1

Due: Noon on Friday, 29 March

1. Consider the following exchange economy. There are two goods (1 and 2) and two consumers (A and B). Preferences and endowments are as follows:

$$u^A\left(x_1^A,\, x_2^A\right) \;=\; x_1^A x_2^A \qquad \overline{x}^A = (0,2)$$

$$u^B\left(x_1^B,\, x_2^B\right) \;=\; \min\left\{x_1^B, x_2^B\right\} \qquad \overline{x}^B = (4,0)$$

   (a) Draw an Edgeworth Box diagram to depict this economy. Your diagram should be clearly labelled, and should include the autarkic allocation as well as a couple of indifference curves for each consumer. (Indifference curves for A do not need to be precisely accurate but those for B should be.)

   [2 marks]

   (b) Identify the set of Pareto efficient allocations and indicate these in your diagram.

   [4 marks]

   (c) Let π denote the price ratio p_1/p_2.

      i. Compute each consumer's optimal consumption of good 1 and optimal consumption of good 2. (These may be functions of π.)

      [4 marks]

      ii. What is the Walrasian equilibrium value for π? Illustrate the associated budget line and Walrasian equilibrium allocation (WEA) in your Edgeworth Box diagram.

      [3 marks]

   (d) Suppose B's endowment were $\overline{x}^B = (10,0)$.

      i. What is the equilibrium value for π in this case?

      [3 marks]

      ii. Indicate the new equilibrium budget line and WEA in an Edgeworth Box diagram.

      [2 marks]

iii. Show that B's utility at the new WEA is exactly the same as it was in the WEA for part (c)(ii), despite the increase in his wealth, but that A's utility has increased despite her wealth not having changed.

[1 mark]

iv. Can you give an intuitive explanation for (iii)?

[1 mark]

2. Xuan-Thuc is a taxpayer in a country with a flat tax rate of t. Her income for the current year is \$1000. Therefore, if she declares her full income she will pay $\$1000t$ in tax, leaving her with $\$1000\,(1-t)$ in post-tax income. If she chooses to declare less than her full income her fraud will be discovered if and only if she is audited by the tax authority, which happens with probability p. If an audit discovers fraud, Xuan-Thuc must pay tax at the penalty rate T on any undeclared income, where $T > t$. For example, if Xuan-Thuc declares income of \$500 then with probability $1-p$ she gets away with this fraud and has post-tax income of

$$\$\left[500\,(1-t)+500\right]$$

and with probability p she is caught and ends up with post-tax income of

$$\$\left[500\,(1-t)+500\,(1-T)\right].$$

Let X denote the amount of income declared by Xuan-Thuc. Therefore, $0 \leq X \leq 1000$ and Xuan-Thuc can choose any value of X in this range.

(a) Let s_1 denote the state in which Xuan-Thuc is audited and s_2 the state in which she is not. Thus, s_1 has probability p and s_2 has probability $1-p$. Let x_1 denote Xuan-Thuc's post-tax income in state s_1 and let x_2 denote her post-tax income in state s_2. Draw a HY diagram with x_1 measured on the horizontal axis and x_2 measured on the vertical axis. In your diagram indicate the points corresponding $X = 0$ and $X = 1000$.

[3 marks]

(b) Xuan-Thuc's "budget line" is the line segment joining the two points that you identified in (a). Using your diagram, calculate the slope of this line and verify that this slope is equal to

$$-\frac{t}{(T-t)} \qquad (*)$$

[1 mark]

(c) Suppose Xuan-Thuc is an expected utility maximiser and she is *risk averse* (i.e., her vNM utility function is strictly concave). If $t = \frac{1}{4}$ and $p = \frac{1}{3}$ what is the *lowest* value of T that will guarantee Xuan-Thuc declares her full income? Be sure to explain your reasoning.

[3 marks]

(d) Suppose Xuan-Thuc has vNM utility function $v(x) = x^2$ (so she is *risk seeking*). If $t = \frac{1}{4}$, $p = \frac{1}{3}$ and $T = \frac{3}{4}$ what value of X will Xuan-Thuc choose? Illustrate her optimal decision by adding an indifference curve through the optimal point in your HY diagram.

*[**Hint:** Recall that risk-seekers have indifference curves that bend the "wrong" way so Xuan-Thuc will choose one end of her "budget line" or the other. Your task is to figure out which one!]*

[3 marks]